STOCHASTIC MODELFOR ALLOCATION OF PERSONNEL IN A MULTI-DEPARTMENTAL MANPOWER SYSTEM

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ABSTRACT

The present paper proposes a homogenous Poisson model for optimum allocation of personnel in a multi-departmental manpower system. In particular, we suggest a model, which considers closed departmental group of employees on various grade levels in different careers from sections of organizations. The objective is to satisfy the set requirements for personnel on each grade level and per department. The numerical illustration of the model shows that personnel requirements were met in most grades within the departmental group. However, cases such as understaffing and overstaffing on some grade levels within the departmental group were also pinpointed for management decision. The model also provides manpower planners with a mild control guide (indicators) on personnel transfer between organizational sections or units. The use of Poisson model is enhanced by a set of programme written in Python.

Key Words: Poisson Model, multi-departmental manpower system, optimal, transfer and overstaffing/ understaffing.

1.0 Introduction

Manpower planning, also called personnel planning seeks to make the utmost use of human resources. This involves the attempt to bring into a balance the demand and supply of people with various knowledge, skills, and qualifications. A Manpower structure can conveniently be described as a dynamic system of stocks and flows. Stocks are the number of people in various classes of manpower system. The flows represent movements of people which take place between classes within the system (i.e. promotions and transfers); from the system to its outside environment (i.e. wastage); and from the outside environment to the system (i.e. recruitment).

Markov models have been used extensively in the area of statistical manpower planning and control. In time homogenous Markov modeling, the transition probabilities from one grade to another are independent of the time (Bartholomew et al, 1991; Guerry and De Feyter, 2009; Dimitriou et al., 2015). In the non-homogeneous Markov models, the transition probabilities vary with time (Dimitriou et al., 2013, Vassiliou, P.C.G., 1998; Vassiliou, P.C.G., 2015). In the Semi-Markov approach, matrix of the transition probabilities are associated with a conditional distribution of duration in a state (Yadavalli and Natarajan, 2001; Vassiliou and Papadopoulou, 1992).

For a manpower system, one of the goals can certainly be to achieve the desired personnel structure in an optimal way (e.g. Bartholomew et al, 1982; Guerry and De Feyter, 2012; Dimitriou et al., 2013; Ossai and Uche, 2009; Kamarudin et al., 2015). Prior work considers problem of employees in which the usual form of progression is "up or out" in a single system (an organization). However, manpower system can be made up of organizations rather than a single one and the staff can move between them. There are few works towards such studies.

The present paper proposes a homogenous Poisson model for optimum allocation of personnel in a multi-departmental manpower system. In the next part we start by providing; the model description; the elements of the multi-departmental systems and the structure for the investigation of mobility within a section (intra-section transitions) and between similar sections (intra- section transitions) of the multi-departmental manpower system. The object model of this study with its parameters is presented in part 3. We focus attention to optimization and write a set of programme in Python in order to obtain adequate numbers of personnel required in part 4. The last part is continued with a numerical application which summarizes the practical results of the model.

2.0 The Model Description

We consider a multi-departmental manpower system in which there are g-th sections and i-th department; g = 1, 2,…y; i = 1,2, … u. For any g-th sections and i-th department, let the states of the grades be denoted by k such that k = 1, 2, … v, with v standing for the top hierachical grade. The intra transitions entail promotion only to the next more senior grade in g-th section in any i-th department. The inter-transfer of personnel between any two sections in any i-th department is allowed. We assume that the g-th sections have the same configuration in terms of grade structure. Recruitment may take place in lower grade. Demotions are not allowed, and wastage occurs in all grade. The unit of time is considered as one year. It is assumed that intra-transition (promotion), inter-transfer and recruitment are under the control of management. All promotions occur at the end of the year.

By proposing Poisson model for the dynamics of the manpower system, the objective is to find probability of personnel lack in a department which fails to satisfy given manpower needs in each department and each grade over a specified planning horizon.The personnel requirements for each department and each grade may be specified in terms of lower and upper bounds. When the probability of total numbers of personnel in each grade level per department is within the lower and upper bound, then the transition probability of the departmental manpower system is considered stable. When it is below lower bound or above upper bound, it is considered unstable, then, one may look at instability. The instability in this case may be that there is either the workforce on a particular grade level or department is low (understaff); there is need for additional input (recruitment) or the workforce is high (overstaff); there is need for right sizing.

Similarly, when the lower bound is exceeded, it may be that an individual may go out in terms of transfer but to another section or unit within the department but such an individual is still within the system. When the upper bound is exceeded it may be that the system has more than enough personnel, the possibility of such is redundancy which can be handled by management through redundant or exit policies.

2.1 Elements of the Multi-departmental Manpower System Model

Notation and Definition:

$s_{g,(k)}(t)$: represents the number of personnel in grade k, section g at time t;

$s_{i,(k)}(t)$: represents the number personnel in grade k, department i at time t;

$S_d(t) = \sum_{i=1}^{u}\sum_{k=0}^{v} s_{i,(v)}(t)$ **:** represents the overall number of categories of personnel in the system at time t.

$p_{g(kl)}$: represents the intra transition probabilities, from one grade to another in section g

$p_{gh,(kl)}$: represents the inter transition probabilities, from grade *k* in section *g* to another grade *l* in section *h*

$p_i = \{p_{g,(kl)} + p_{gh,(kl)}\}$: the merger of intra and inter transition probabilities, from one grade to another in department *i* at time t

$r_{i,(k)}$: is the probability of distribution of new recruits into grade i department i.

Using the above notations and definitions for the elements of the multi-departmental system. An individual employee in the manpower system is allocated to a particular grade level in a section within each department. Since it is presumed that demotions are not allowed and that transitions may only be made to the next higher grade in a department. It is expected that employees move up from one grade to another, i.e. from grade k to grade l, the topmost grade, grade v loses personnel only by wastage and the lower grades, grade k =1, 2, and 3 gain staff through recruitment. Intra (promotion) and inter (transfer) transitions are controlled by management. Since the numbers of people in each department and per grade, at the beginning of year t are variables, we can conveniently fit Poisson model for the multi-departmental manpower structure. We note that in practice, it is not likely that there will be an absolute specification for the numbers of personnel in each department and per grade to remain constant from year to year, it may be possible to be pre-specified that the numbers of personnel required must not vary outside limits. In these situations, the optimal number required may be pre-specified by management.

2.2. The Structure of Multi-departmental Transition Matrix

2.2.1 Intra-sectional Transition Probability

With respect to section *g,* the intra transition probabilities, from one grade to another in section *g* are characterized by the probability $p_{g(kl)}$. The intra transition matrix with respect to section *g* is the matrix $A_g = \{p_{g(kl)}\}$:

$$A_g = \begin{bmatrix} p_{g.(11)} & p_{g,(12)} & \cdot & \cdot & \cdot & p_{g,(1v)} \\ p_{g.(21)} & p_{g.(22)} & \cdot & \cdot & \cdot & p_{g.(2v)} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ p_{g.(v1)} & p_{g.(v2)} & \cdot & \cdot & \cdot & p_{g.(vv)} \end{bmatrix} \quad (1)$$

2.2.2 Inter-sectional Transition Probability

For employees from grade *k* in section *g* to another grade *l* in section *h* the inter transition probability is denoted by $\{p_{gh,(kl)}\}$. The inter-transitions from section *g* to section *h* is the matrix $A_{gh} = \{p_{gh,(kl)}\}$;

$$A_{gh} = \begin{bmatrix} p_{gh.(11)} & p_{gh,(12)} & \cdot & \cdot & \cdot & p_{gh,(1v)} \\ p_{gh.(21)} & p_{gh.(22)} & \cdot & \cdot & \cdot & p_{gh.(2v)} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ p_{gh.(v1)} & p_{gh.(v2)} & \cdot & \cdot & \cdot & p_{gh.(vv)} \end{bmatrix} \quad (2)$$

2.2.3 Departmental Transitional Probability

With respect to department *i* the merger of intra and inter transition probabilities, from one grade to another in department *i* at time t are characterized by the probabilities $p_i = \{p_{g,(kl)} + p_{gh,(kl)}\}$. The departmental transition matrix with respect to department *i* is the matrix $P_i = \{A_g + A_{gh}\}$:

$$P_i = \begin{bmatrix} p_{g,(11)} + p_{gh,(11)} & p_{g,(12)} + p_{gh,(12)} & \cdot & \cdot & \cdot & p_{g,(1v)} + p_{gh,(1v)} \\ p_{g,(21)} + p_{gh,(21)} & p_{g,(22)} + p_{gh,(22)} & \cdot & \cdot & \cdot & p_{g,(2v)} + p_{gh,(2v)} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ p_{g,(v1)} + p_{gh,(v1)} & p_{g,(v2)} + p_{gh,(v2)} & \cdot & \cdot & \cdot & p_{g,(vv)} + p_{gh,(vv)} \end{bmatrix} \quad (3)$$

This can be written as:

$$P_i = \begin{bmatrix} p_{i,(11)} & p_{i,(12)} & . & . & . & p_{i,(1v)} \\ p_{i,(21)} & p_{i,(22)} & . & . & . & p_{i,(2v)} \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ p_{i,(v1)} & p_{i,(v2)} & . & . & . & p_{i,(vv)} \end{bmatrix} \quad (4)$$

It follows that

$$\sum_{l=1}^{v} p_{i,(kl)} = \sum_{l=1}^{v} p_{i,(kl)} + \sum_{1}^{v} p_{gh,(kl)} = 1$$

For all departments $i = 1, 2, .., u$ the information on the departmental transitions probabilities are gathered in the block diagonal matrix P_d **:**

$$P_d = \begin{bmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_2 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & P_u \end{bmatrix} \quad (5)$$

The matrix P_d will be called the **departmental transition matrix**.

3.0 The Poisson Model

Proposition 1: Let $X_1(t)$ represents the number of arrival times of an employee from one grade to another in section g. Let $X_2(t)$ represents the number of arrival times of an employee from section g into another section h. The intra and inter transition distributions of $X_1(t)$ and $X_2(t)$ have Poisson distributions with parameters λ_1 and λ_2, respectively. Then the sum $\lambda_1 + \lambda_2$ has a Poisson distributions with parameter $\lambda_1 + \lambda_2$

Proof: By law of total probability

$$P[X(t) = s] = \sum_{q=0}^{s} [P(X_1(t) = q) \quad (P(X_2(t) = s - q)] \tag{6}$$

$$= \sum_{q=0}^{s} \frac{e^{-\lambda_1 t} (\lambda_1 t)^r}{q!} \frac{e^{-\lambda_2 t} (\lambda_2 t)^{s-q}}{q! \; (s - q)} \tag{7}$$

$$= \frac{e^{-(\lambda_1 + \lambda_2)t}}{s!} \sum_{q=0}^{s} \frac{s!}{q! \; (s - q)} (\lambda_1 t)^q (\lambda_1 t)^{s-q}$$

$$= \frac{e^{-(\lambda_1 + \lambda_2)t}}{s!} \sum_{q=0}^{s} {}^sC_q \; (\lambda_1 t)^q (\lambda_1 t)^{s-q} \tag{8}$$

$$= \frac{e^{-(\lambda_1 + \lambda_2)t}}{s!} (\lambda_1 t + \lambda_1 t)^s$$

$$= \frac{e^{-(\lambda_1 + \lambda_2)t}}{s!} ((\lambda_1 + \lambda_1)t)^s \tag{9}$$

$X(t) = X_1(t) + X_2(t)$ is a Poisson distribution with parameter $\lambda_1 + \lambda_1$

Definition: Let the number of arrival into the state of manpower system in time (0, t] be represented by $X(t)$. Define, for $\tau < t$,

$$P_{kl}(\tau, t) = P[X(t) = l / X(\tau) = k]$$

Further let the number of arrival occur under the following requirements:

1. The number of arrival into the state of manpower system occur in non-overlapping interval of time are independent of each other.
2. For sufficiently small Δt, there is a constant λ such that the probabilities of arrival occur in the interval $(t, t + \Delta t)$ are given as follows:
    a. $P_{kk}(t, t + \Delta t) = 1 - \lambda \Delta t + 0(\Delta t)$
    b. $P_{k,k+1}(t, t + \Delta t) = \lambda \Delta t + 0(\Delta t)$
    c. $\sum_{l=k+2}^{\infty} P_{kl}(t, t + \Delta t) = 0(\Delta t)$
    d. $P_{kl}(t, t + \Delta t) = 0 \quad l < k$ (10)

Proposition 2: (Bhat 1984) Assume for the multi-departmental manpower system that $X(t)$ represent the number of times an employee arrive into each grade of the departmental manpower system. The transition distribution of the multi-departmental manpower system $X(t)$ has a Poisson distribution given by

$$P_d(t) = P[X(t) = s / X(0) = 0] = e^{-\lambda t} \frac{(\lambda t)^s}{s!} \quad s = 0,1,... \qquad (11)$$

3.1 The Criterion for manpower requirements

We consider the question of estimating the probability that a given departmentalized and grade manpower requirement satisfies a threshold at a unit time. In attempt to do so, we follow the approach by Adell and Jodra (2005) under the monotonicity of certain sequences involving tail probabilities of Poisson distribution and central limit theorem that the sequence $(S_s \le s),\ s \in \mathbb{N})$ and $(S_s \ge s),\ s \in \mathbb{N})$ strictly decrease to $\frac{1}{2}$, and Median $(S_s) = s$, to obtain the median value for the maximum and minimum numbers required in each grade and per department.

Let:
$s_{i,(Uk)}$ represents the maximum numbers (upper bound) of staff required in each grade per department,
$s_{i,(Lk)}$ represents the minimum number (lower bound) of staff required in each grade per department and
$s_{i,Mk}$ represents the median value for the maximum and minimum numbers required in each grade and per department.

The criterion satisfying the optimal number of staff required in each grade and per department can be expressed as

$$\text{Probability of } (s_{i,(k)} > s_{i,Lk}) = s_{i,Mk} \qquad (28)$$

We write a program in Python for optimization of staff allocation.

4.0 The Python code for Poisson model optimization.

```
import csv
import glob
import math
# import numpy
# from scipy.stats.distributions import poisson
# transfer --- btw
# n less than minimun --- under-staff
```

```
def main():
    no_of_year = 7
    dir = "data/"
    mean_data_dir = dir + "mean/data.csv"
    mean_list = read_csv_file_content(mean_data_dir)
    min_data_dir = dir + "min/data.csv"
    min_write_path = dir + "min/"
    # print mean_list
    for x in range(no_of_year):
        filename = dir + "year" + str(x+1) + "/data.csv"
        data_list = read_csv_file_content(filename)
        # print data_list
        generated_poission_data = generatePoisionData(data_list, mean_list)
        # print generated_poission_data
        write_path = dir + "year" + str(x+1) + "/"
        write_poission_data(generated_poission_data, write_path)
    min_data = read_csv_file_content(min_data_dir)
    generated_poission_data = generatePoisionData(min_data, mean_list)
    write_poission_data(generated_poission_data, min_write_path)
    ### -------->
    #++ compare
    min_poisson_data = generated_poission_data
    max_poisson_data = read_csv_file_content(dir + "max/poisson.csv")
    for x in range(no_of_year):
        filename = dir + "year" + str(x+1) + "/poisson.csv"
        data_list = read_csv_file_content(filename)
        # print data_list
        # print data_list
        compared_data = compare_min_max_and_data(data_list, min_poisson_data,
max_poisson_data)
        write_path = dir + "year" + str(x+1) + "/"
        write_compared_data(compared_data, write_path)
def compare_min_max_and_data(data_list, min_poisson_data, max_poisson_data):
    row = len(data_list)
    col = len(data_list[0])
    new_data = list()
    for x in range(row):
        # print x, data_list[x]
        new_row = list()
        for y in range(col):
            # print y
            if(data_list[x][y] == '' or (data_list[x][y] == '0' and data_list[x][y] == '0')):
                val = ''
```

```
                new_row.append(val)
            else:
                a = float(data_list[x][y])
                b = float(min_poisson_data[x][y])
                c = float(max_poisson_data[x][y])
                print x, y, a, b, c
                if(a == b):
                    val = 'OPTIMAL'
                else:
                    val = 'NON-OPTIMAL'
                if (y == col-1):
                    if(a > c):
                        val = 'OVERSTAFF'
                    elif(a == c):
                        val = 'OPTIMAL'
                    elif(a < b):
                        val = 'UNDERSTAFFED'
                    else:
                        val = 'TRANSFER'
                print a, b, c, val
                new_row.append(val)
        new_data.append(new_row)
    return new_data

def read_csv_file_content(file_name):
    try:
        with open(file_name, 'rb') as f:
            reader = csv.reader(f)
            return list(reader)
    except:
        print 'Error while opening the file'
        return []
def poisson_cummulative_probability(actual, mean):
    g = 0
    for z in range(actual):
        p = math.exp(-mean)
        for i in xrange(z+1):
            p *= mean
            p /= i+1
        g = g + p
    return g
def generatePoisionData(main_data, mean_data):
    row = len(main_data)
```

```
    col = len(main_data[0])
    new_data = list()
    for x in range(row):
        new_row = list()
        for y in range(col):
            if(main_data[x][y] == '' or (main_data[x][y] == '0' and mean_data[x][y] == '0')):
                prob = ''
                new_row.append(prob)
            else:
                a = int(main_data[x][y])
                b = int(mean_data[x][y])
                prob = poisson_cummulative_probability(a, b)
                prob = round(prob, 6)
                new_row.append(prob)
        new_data.append(new_row)
    return new_data
def write_poission_data(data, path):
    print "\nWriting results into file ..."
    with open(path + "poisson.csv", "w") as f:
        writer = csv.writer(f, delimiter=',')
        writer.writerows(data)
    print "\Finish Writing 100%"
def write_compared_data(data, path):
    print "\nWriting results into file ..."
    with open(path + "result.csv", "w") as f:
        writer = csv.writer(f, delimiter=',')
        writer.writerows(data)
    print "\Finish Writing 100%"
if __name__ == "__main__":
    main()
```

5.0 Discussion:

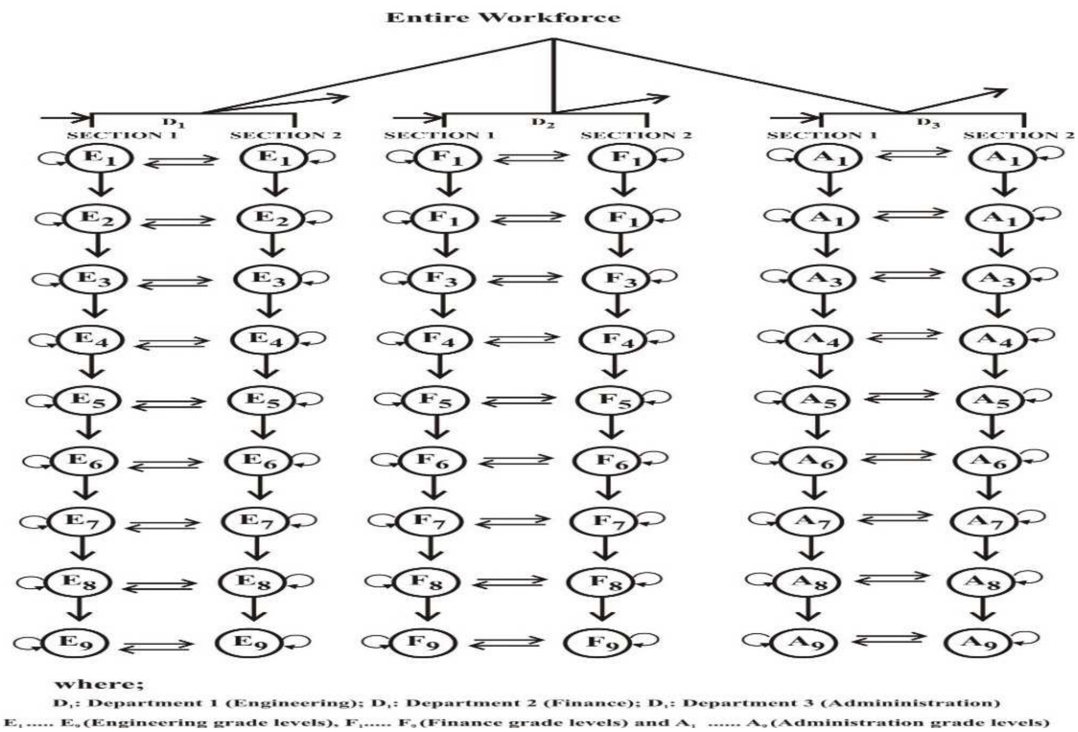

Figure 1. The Multi-departmental Workforce Structure

We applied Poisson model on the manpower data of two ministries in the Federal Civil Service, Abuja. Three departments were considered; Engineering Services Department, Finance Department and Administrative Department. Employees on similar career path were grouped together to form what we describe as department. Each department is divided into sections. The employees in each section are representative of each ministry. The transfer of employees between sections within each department is allowed. There is no movement of employees between any two departments. The rank of officers in each department is divided into nine grade levels; the officer cadre, grade level 1- 3, middle management cadre, grade level 4-6, the senior management cadre, grade level 7-9.
The initial numbers of staff allocated on each grade, maximum numbers of staff required on each grade, the mean obtained, the maximum probability required in each grade, the minimum probability required in grade and Poisson median value for three departments of the manpower data are presented in tables 1-3

Department	Grade	Initial numbers allocated per grade	Maximum numbers in grade	Minimum numbers in grade	Mean	Maximum probability in grade	Minimum probability in grade
Engineering Dept.	1	96	96	90	96	0.61	0.29
	2	84	86	82	86	0.61	0.36
	3	72	72	68	72	0.62	0.35
	4	55	56	52	56	0.64	0.33
	5	52	52	48	52	0.64	0.32
	6	40	41	38	41	0.66	0.36
	7	30	30	27	30	0.68	0.33
	8	9	10	8	10	0.70	0.33
	9	2	3	1	3	0.82	0.20
Total		**440**	**446**	**414**	**446**	**0.78**	**0.07**

Table 1. Manpower data for Engineering Department

Department	Grade	Initial numbers allocated per grade	Maximum numbers in grade	Minimum numbers in grade	Mean	Maximum probability in grade	Minimum probability in grade
Finance Dept.	1	45	49	44	46	0.70	0.42
	2	41	45	40	42	0.71	0.42
	3	34	37	33	35	0.67	0.41
	4	28	30	25	29	0.76	0.26
	5	18	20	16	20	0.73	0.22
	6	14	15	12	15	0.84	0.27
	7	9	8	9	11	0.59	0.34
	8	4	4	3	5	0.82	0.27
	9	1	2	0	2	0.68	0.14
Total		**194**	**211**	**182**	**205**	**0.89**	**0.06**

Table 2. Manpower data for Finance Department

Department	Grade	Initial numbers allocated per grade	Maximum numbers in grade	Minimum numbers in grade	Mean	Maximum probability in grade	Minimum probability in grade
Administrative Dept.	1	54	56	52	54	0.64	0.43
	2	44	48	41	45	0.71	0.31
	3	38	42	36	40	0.66	0.30
	4	29	30	27	29	0.62	0.40
	5	22	24	20	22	0.71	0.39
	6	19	22	18	20	0.72	0.38
	7	10	12	8	11	0.69	0.23
	8	4	6	2	5	0.76	0.12
	9	1	2	0	2	0.68	0.14
Total		**221**	**242**	**204**	**228**	**0.83**	**0.06**

Table 3. Manpower data for Administrative Department

The results for the application of the model at the initial period t = 0 for the manpower data are shown in tables 4 - 6. For the purpose of illustration, in the Engineering department (table 4) the result for Poisson probability of 96 numbers of staff allocated in the officer cadre of grade level 1 is 0.5. This probability when compared with minimum probability required in the grade and the Poisson median value (optimal value) of 0.5. The result indicates that the Poisson probability of (0.5) for the total numbers of staff at grade level 1 in the department exceeds the minimum probability required of 0.3 in the grade and also equals threshold optimality of 0.5. Hence, it is considered optimal. The result is the same for total numbers of personnel in grade levels 3, 4,5,6,7 and 8. This implies that the numbers of employees on the stated categories satisfied the manpower requirements for the grade levels.

Department	**Grade**	**Numbers allocated per grade**	**Poisson probability in grade**	**Minimum probability in grade**	**Poisson Median Value**	**Result from the Model**
Engineering Services Dept.	1	96	0.5	0.3	0.5	OPTIMAL
	2	84	0.4	0.3	0.5	TRANSFER
	3	72	0.5	0.3	0.5	OPTIMAL
	4	55	0.5	0.3	0.5	OPTIMAL
	5	52	0.5	0.3	0.5	OPTIMAL
	6	40	0.5	0.4	0.5	OPTIMAL
	7	30	0.5	0.3	0.5	OPTIMAL
	8	9	0.5	0.3	0.5	OPTIMAL
	9	2	0.4	0.2	0.5	TRANSFER

Table 4

The result for total numbers of staff allocated to grade level 2 and 9 indicate need for personnel TRANSFER. The Poisson probability (0.4) of staff allocated in grade level 2 is above the minimum required (0.3) but below the median value (0.5). This indicates a trigger for the manpower planner on personnel transfer distribution in grade level 2 between two sections within the Engineering department.

Department	**Grade**	**Numbers allocated per grade**	**Poisson probability in grade**	**Minimum probability in grade**	**Poisson Median Value**	**Result from the Model**
Finance Dept.	1	45	0.5	0.4	0.5	OPTIMAL
	2	41	0.5	0.4	0.5	OPTIMAL
	3	34	0.5	0.4	0.5	OPTIMAL
	4	28	0.5	0.3	0.5	OPTIMAL
	5	18	0.4	0.2	0.5	TRANSFER
	6	14	0.5	0.3	0.5	OPTIMAL
	7	9	0.2	0.3	0.5	UNDERSTAFF
	8	4	0.4	0.3	0.5	TRANSFER
	9	1	0.4	0.1	0.5	TRANSFER

Table 5

The result in table 5 for manpower stocks in Finance department shows levels of optimality for total numbers of personnel in grade level 1,2,3,4 and 6. There is a critical situation of understaff in grade level 7. There is need for personnel transfer between sections within the Finance department in grade levels 5, 7 and 8.

Department	Grade	Numbers allocated per grade	Poisson probability in grade	Minimum probability in grade	Poisson Median Value	Result from the Model
Administrative Dept.	1	54	0.5	0.4	0.5	OPTIMAL
	2	44	0.5	0.3	0.5	OPTIMAL
	3	38	0.4	0.3	0.5	TRANSFER
	4	29	0.5	0.4	0.5	OPTIMAL
	5	22	0.6	0.4	0.5	OVERSTAFF
	6	19	0.4	0.4	0.5	TRANSFER
	7	10	0.5	0.2	0.5	OPTIMAL
	8	4	0.4	0.1	0.5	TRANSFER
	9	1	0.4	0.1	0.5	TRANSFER

Table 6

The Administrative department table 6 indicates optimal level for personnel in grade level 1, 2 and 7; needs for personnel transfer in grade level 3, 6, 8 and 9; while personnel in grade level 5 exceed the specified threshold optimal requirement, hence the personnel in grade level are overstaffed.

The results for other planning horizons are shown in tables 7 - 12. The manpower data for engineering, finance and administrative departments at time t=3 show some levels of optimality in the all cadres for all the three departments. This indicates that the transition probabilities for the entire multi-departmental structure in the period is relatively stable. We observe lack of personnel at time t= 1 in grade level 6 in department of engineering, at time t= 2 in the grade level 5 in Finance department, at time t= 4 in grade level 7, 8 in engineering department, at time t= 5 in grade level 4 in administrative department and time t= 6 in grade level 1, 6 in administrative department. The short fall in these departments are to be filled with new recruit within the total maximum and minimum numbers required in each grade level.

	Engineering Dept.		Finance Dept.		Admin Dept.	
Period t=1	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result
1	94	TRANSFER	46	OPTIMAL	54	OPTIMAL
2	88	OVERSTAFF	42	OPTIMAL	44	OPTIMAL
3	71	OPTIMAL	35	OPTIMAL	39	OPTIMAL
4	55	OPTIMAL	28	OPTIMAL	28	OPTIMAL
5	51	OPTIMAL	19	OPTIMAL	21	OPTIMAL
6	37	UNDERSTAFF	14	OPTIMAL	19	OPTIMAL
7	29	OPTIMAL	10	OPTIMAL	10	OPTIMAL
8	9	OPTIMAL	5	OVERSTAFF	4	TRANSFER
9	2	TRANSFER	1	TRANSFER	1	TRANSFER

Table 7

	Engineering Dept.		Finance Dept.		Admin Dept.	
Period t=2	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result
1	95	OPTIMAL	45	OPTIMAL	53	OPTIMAL
2	86	OPTIMAL	42	OPTIMAL	44	OPTIMAL
3	72	OPTIMAL	34	OPTIMAL	42	OVERSTAFF
4	55	OPTIMAL	28	OPTIMAL	28	OPTIMAL
5	51	OPTIMAL	19	OPTIMAL	21	OPTIMAL
6	40	OPTIMAL	14	OPTIMAL	19	OPTIMAL
7	30	OPTIMAL	5	UNDERSTAFF	10	OPTIMAL
8	9	OPTIMAL	4	TRANSFER	4	TRANSFER
9	2	TRANSFER	2	OVERSTAFF	1	TRANSFER

Table 8

	Engineering Dept.		Finance Dept.		Admin Dept.	
Period t=3	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result
1	94	TRANSFER	46	OPTIMAL	54	OPTIMAL
2	86	OPTIMAL	41	OPTIMAL	44	OVERSTAFF
3	72	OPTIMAL	34	OPTIMAL	40	OPTIMAL
4	55	OPTIMAL	28	OPTIMAL	28	OPTIMAL
5	51	OPTIMAL	19	OPTIMAL	21	OPTIMAL
6	40	OPTIMAL	14	OPTIMAL	19	OPTIMAL
7	29	OPTIMAL	10	OPTIMAL	10	OPTIMAL
8	9	OPTIMAL	4	TRANSFER	4	TRANSFER
9	2	TRANSFER	1	TRANSFER	1	TRANSFER

Table 9

	Engineering Dept.		Finance Dept.		Admin Dept.	
Period t=4	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result
1	91	TRANSFER	46	OPTIMAL	54	OPTIMAL
2	83	TRANSFER	41	OPTIMAL	45	OPTIMAL
3	71	OPTIMAL	35	OPTIMAL	40	OPTIMAL
4	55	OPTIMAL	28	OPTIMAL	28	OPTIMAL
5	51	OPTIMAL	20	OVERSTAFF	21	OPTIMAL
6	40	OPTIMAL	14	OPTIMAL	19	OPTIMAL
7	27	UNDERSTAFF	10	OPTIMAL	10	OPTIMAL
8	7	UNDERSTAFF	4	TRANSFER	4	TRANSFER
9	2	TRANSFER	2	OVERSTAFF	1	TRANSFER

Table 10

	Engineering Dept.		Finance Dept.		Admin Dept.	
Period t=5	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result
1	97	OVERSTAFF	46	OPTIMAL	53	OPTIMAL
2	86	OPTIMAL	42	OPTIMAL	44	OPTIMAL
3	72	OPTIMAL	35	OPTIMAL	39	OPTIMAL
4	55	OPTIMAL	28	OPTIMAL	27	UNDERSTAFF
5	51	OPTIMAL	19	OPTIMAL	21	OPTIMAL
6	40	OPTIMAL	14	OPTIMAL	20	OVERSTAFF
7	30	OPTIMAL	10	OPTIMAL	10	OPTIMAL
8	9	OPTIMAL	4	TRANSFER	4	TRANSFER
9	2	TRANSFER	1	TRANSFER	1	TRANSFER

Table 11

	Engineering Dept.		Finance Dept.		Admin Dept.	
Period t=6	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result	Numbers of Staff per grade	Optimization Result
1	94	TRANSFER	45	OPTIMAL	52	UNDERSTAFF
2	85	OPTIMAL	41	OPTIMAL	44	OPTIMAL
3	72	OPTIMAL	34	OPTIMAL	39	OPTIMAL
4	55	OPTIMAL	28	OPTIMAL	28	OPTIMAL
5	51	OPTIMAL	20	OVERSTAFF	21	OPTIMAL
6	40	OPTIMAL	13	TRANSFER	14	UNDERSTAFF
7	29	OPTIMAL	10	OPTIMAL	10	OPTIMAL
8	9	OPTIMAL	4	TRANSFER	4	TRANSFER
9	2	TRANSFER	2	OVERSTAFF	1	TRANSFER

Table 12

5.1 Conclusion:

In this paper we suggested a model for hierarchical manpower system that considers employees in similar job functions from different subgroup of manpower systems. The merger of these subgroups of employees constitute departments. We assume Poisson distribution for both intra and inter transition distributions. The objective of this model is to satisfy the set requirements for personnel on each grade level and per department in order to determine the probability of personnel lack. A set of programme is written in Python for computation analysis of the model to determine optimum allocation of personnel on each grade level and per department. The numerical illustration indicates optimum acceptable personnel, the overstaff (surplus) and under staff (shortages). The under-staff is recommended for recruitments and overstaff is to be handled by manager in line with the organizations exit policy. The model also provides manager with a mild control guide (indicators) on personnel transfer between organizational sections or units.

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