Analyzing the Diversification in the Optimal Portfolio Choice: an empirical study of the Moroccan Stock Market

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Abstract

The Financial markets in the emerging countries in Asia, Latin America and Eastern Europe have sparked an important literature aimed at understanding how they work, their organization and future prospects, and the different techniques and methods of allocation of their portfolios. However, few studies have been devoted to the Moroccan financial market; especially those concerned with the problem of diversification in the optimal portfolio choice.

In this study, we will try to understand the portfolios wide diversification strategy that is based on the correlation of financial assets in the Moroccan financial market. Similarly, this paper seeks to present a critical analysis of this strategy of the optimal diversification and implement the mixed linear program with absolute deviation model of Konno & Yamazaki Simplified by Hamza & Janssen. This program will reduce the size of the portfolio to be optimized in order to avoid the management and transaction expenses of the financial assets generated by this strategy.

Key Words: diversification, costs of transactions, absolute mean-deviation model simplified, mixed linear program, Casablanca Stock market.

Classification (JEL): G11, G17, C61

1. Introduction

The choice of an optimal portfolio of the financial assets has been, for a long time, a topic of a primary interest in the financial field. The basic idea is to maximize the return of the portfolio while minimizing its risk. In this context, several theoretical models of the assets allocation have been studied since the early 1950s with the aim of resolving this problem of the portfolio choice.

The article of the economist Harry Markowitz to the United States published in 'The Journal of Finance' in 1952 launched the early developments of the modern theory of portfolio management. After it had been limited to the academic field for a long time, this theory was eventually imposed on the professionals of the financial world. Markowitz suggested expressing a security's interest by the expectation of its returns and the risk by its variance. The investor, for the construction of his portfolio, will seek to make a full return with a minimum risk. This approach, called the mean-variance approach, led him to win the Nobel Prize in 1990.

He has also introduced the concept of the efficient border deducted from the minimum variance portfolio for a given expectation of return, which represents the optimal combination of risk and return. The optimization is done by defining a function of utility that represent the investors' preference taking into account their aversion to the risk and maximizing the one, which is given the constraint represented by the efficient border.

The works of Harry Markowitz helped, as well, in establishing a theory of the optimal diversification of the stock market portfolio. In fact, Markowitz has established the strategy of diversifying his portfolio and reducing its risk level while maintaining a satisfactory return. Furthermore, among his proposals, he suggests the use of all types of assets to achieve a good diversification.

The diversification can occur at several levels: not only securities, sectors, countries, regions and investment types, but also styles and strategies. It can reduce the risk of a portfolio except when the investment components are weakly correlated.

The standard theory of the choice of the financial portfolio postulates that the investor arbitrates among all the assets that exist in the market so as to maximize the relation return/risk. Thus, it determines a well-diversified and efficient portfolio. This configuration involves transactions costs[1]. Nevertheless, these transaction costs are not taken into consideration; whereas they cannot be ignored in reality.

In this context, Pogue showed in 1970 that the transaction costs could sometimes exceed, and even further, the expected profitability of the investor, especially for the large portfolios.

Similarly, Leland showed in 1985 that some frequent adjustments of the portfolio to keep the assets close to their target proportions, led to very high transaction costs.

Consequently, the reduction in portfolio size before even carrying out its optimization is very useful to avoid heavy expenses due to the transaction costs.

To achieve our goal and deal with our problem, we must give answers to the following derived questions: *What is the necessary threshold of diversification to eliminate the specific risk? How many assets will be sufficient to diversify significantly a portfolio of financial assets? How should we select them so as to optimize the diversification to reach a fixed threshold of assets?*

In this regard, this work will focus mainly on the presence of a constraint that makes the diversification of our portfolio more efficient within the Moroccan financial market. In the financial discipline, it is called the constraint of the minimum threshold of investment. It aims at reducing the rate of assets, which make the optimal portfolio.

This work is organized as follows. After a general introduction. The second section is devoted to the review of literature mainly the principle of diversification and optimal portfolio choice. Indeed it presents a critical analysis of diversification and a program of resolution, namely the mixed linear program formulated by Hamza & Janssen (*simplifying the model of Konno & Yamazaki),* which takes into consideration the constraints of the purchase threshold. In the third one, we illustrate our approach through a digital application using the historic weekly returns of 74 financial securities that build the Moroccan stock market from January 2, 2013 - June 08, 2014. We will rely also on the econometric software E-views 7 for the tests of the different statistical parameters and Matlab software for the simulation of our resolution program. In the last section, we conclude this work with a discussion about the results and a conclusion.

2. Literature Review

2.1. Diversification and optimal choice of a financial assets portfolio.

As illustrated by Clauss (2011), the principle of diversification is based on an old saying: "*Do not put all one's eggs in one basket.*" To diversify means to compose a portfolio of securities from different sectors in order to distribute the overall risk between these different securities. An investor, who cannot take risk, will build a well-diversified portfolio by investing in different assets**.**

2.1.1. The Formulation of diversification and the mean-variance approach.

The pioneer of the modern finance, Markowitz has demonstrated mathematically the reality of diversification. He established that the total risk of a group of securities is less than the sum of risks of

[1] The theory of transactions' costs, founded by Coase in 1937 and particularly elaborated by Williamson since 1975, resulted in a very important empirical development since 1985. In 1991, the Royal Academy in Sweden gave the Nobel Prize of Economy to Ronald H.Coase for the discovery and clarification of the role of the costs of transactions

these individual securities[2]. In other words, investing in a group of securities reduces the rate of risk without losing the return of the portfolios.
The diversification consists in combining several instruments of investment within the same portfolio. According to Harry Markowitz, *"an investor can reduce the risk of his portfolio simply by holding assets that are not or can be positively correlated, thus, diversifying his investment."* Diversification is efficient when the risk is decreased to the maximum, either absolutely or for a given level of return. Thus, the quality of diversification depends on two parameters:

- The number of securities included in the portfolio.
- The level of correlation between the returns of securities.

The selection of a security to include it in a portfolio is not made according to its individual characteristics, but according to its behavior within the portfolio.

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sum_{i=1}^{N} x_i \left[\sum_{j=1}^{N} x_j \sigma_{ij} \right]$$

Thus, the overall risk of the portfolio is the weighted sum of contributions to the risk of each asset. Considering that a uniform weighted portfolio consists of N securities.
The proportion invested in any security is $\left(x_i = {}^{1}/_{N}\right)$. Thus the total risk of a portfolio can be calculated by the variance or standard deviation of its returns:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \Leftrightarrow \sigma_p^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij}$$

That is $\sigma_p^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sigma_{ii} + \frac{1}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sigma_{ij}$

Noting that V is the largest variance. Thus, we shall have $\frac{1}{N^2} \sum_{i=1}^{N} \sigma_{ii} \leq \frac{VN}{N^2} = \frac{V}{N}$

This ratio tends to zero when N strives for the infinity (it is the effect of non-correlation).

So, $\overline{\sigma}_{ij}$ is the average covariance between the securities, thus, the weighted sum of covariance will be: $\frac{1}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sigma_{ij} = \frac{1}{N^2} (N^2 - N) \overline{\sigma}_{ij} = (1 - \frac{1}{N}) \overline{\sigma}_{ij}$

The weighted sum of the covariance tends to the average covariance when the number of securities in the portfolio increases[3]. When increasing the number of securities, the decrease of risk is at first rapid, and then it slows down sharply. Beyond a certain number, it becomes useless to keep diversifying because the marginal benefits of diversification decrease when the portfolio is diversified, while the marginal costs remain high due to transaction costs. The following figure illustrates this result:

[2] H. Markowitz (1959), *Portfolio Selection*: efficient diversification of investment. Yale University Press.
[3] Jacquillat and Solnik '*Les marchés financiers et la gestion de portefeuille*', p.66, Dunod. 1986

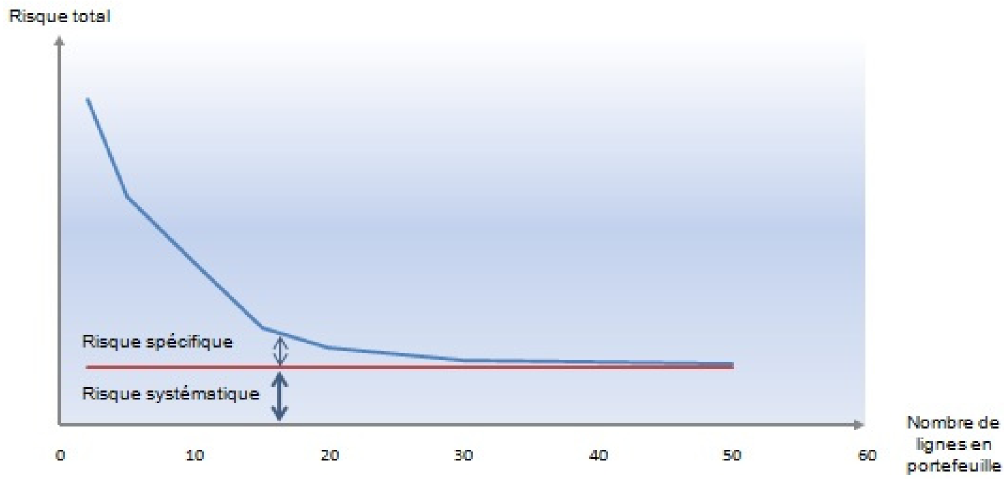


Figure 1: Risk and diversification

Thus, the total risk, which affects the expected return of a value, consists of the systematic and specific risks.

- Systematic or non-diversifiable risk: Systematic risk is attributable to the general movements of the market and economy[4]. This risk cannot be eliminated by diversification.
- Specific or diversifiable risk[5]: The peculiarity of the specific risk is that it can be diversified within one portfolio. It strives for zero when the number of securities in a portfolio is important enough (This is the effect of diversification). That is why it is not remunerated.

In addition, all the empirical studies that have been conducted in some financial markets showed that the return of diversification depends on the structure of covariance[6] and correlation[7] between the securities. Indeed, the specific risk of the portfolio decreases when the number of securities constituting the portfolio increases; by decreasing well as the total risk of the portfolio, until it no longer supports the market risk. The number of securities to hold in its portfolio to achieve a total diversification depends heavily on the correlations between the returns of securities.

2.1.2. The mean absolute deviation approach and diversification

We will explain in the following pages the effect of diversification using the risk function $K(x)$ defined through the function Piece Wise Linear (PWL), instead of the Quadratic function. This is the mean absolute difference of the portfolio return compared with its average. Thus, we consider a uniform weighting portfolio consisting of securities. The proportion invested in any security j ($j = 1, \dots . N$) is $x_j = 1/N$. We have:

$$K(x) = E\left[\left|R(x) - E(R(x))\right|\right] = E\left[\left|\sum_{j=1}^{N} R_j x_j - E(\sum_{j=1}^{N} R_j x_j)\right|\right] = \frac{1}{N} E\left|\sum_{j=1}^{N} R_j - E(\sum_{j=1}^{N} R_j)\right|$$

Konno & Yamazaki showed that their risk measurement and the standard deviation of portfolio returns (the function of risk based Markowitz) are equivalent to a constant if asset returns follow a multivariate normal distribution.

[4] This type of risk is generated from some unexpected macro-economic events like inflation, shock over the rate of interest, higher rate of unemployment, recessions, the change of governments,…, affecting the securities.

[5] The specific risk or micro-risk is linked to the factors which its influence receives on it firm or the group of firms, especially the strikes, changes in the taste of the consumers, errors of management and judicial proceedings.

[6] $\sigma_{ij} = \mathrm{cov}(R_i, R_j) = E[(R_i - E(R_i))(R_j - E(R_j)]$

[7] $\sigma_{ij} = \mathrm{cov}(R_i, R_j) = E[(R_i - E(R_i))(R_j - E(R_j)]$

If the vector of the securities returns $(R_1.........R_n)$ is distributed according to a multivariate normal law, $N(\mu)=(\mu_1,........,\mu_n),\Sigma=(\sigma_{ij})_{1\le i.j\le n}$

Then $R(x)=N(\mu(x)=\sum_{j=1}^{N}\mu_j x_j,\sigma(x)=\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}x_i x_j})$

$K(x)=\sqrt{\frac{2}{\pi}}\sigma(x)$, thus $\lim_{N\to\infty}K(x)=\lim_{N\to\infty}\sqrt{\frac{2}{\pi}\sigma^2(x)}=\sqrt{\frac{2}{\pi}}\bar{\sigma}$

Therefore, the diversification has a limit and the risk, which is measured by the absolute deviation of the portfolio average return not fully diversifiable.

2.2. Critical analysis of diversification

The diversification of a portfolio, if properly conducted, is of real interest. But it also has its own limitations. Diversification helps to reduce the specific risk of the assets, but by no means systematic risk that comes from the market and on which diversification will have no influence.

Similarly, the total diversification is not very realistic for a portfolio manager, especially when the results include minimal amounts of investment. Therefore, this prevents the manager from investing the very low optimal sum provided by the optimization model.

To remedy the presence of small amounts in the optimal diversified portfolio, investors can eliminate securities with a lower proportion of fixed capital at a minimum level, and be content with a portfolio theoretically resulting suboptimal. This is unfortunately one of the major difficulties encountered in practice by managers' portfolios.

To account for these practical aspects, it is interesting to study whether risk diversification can be achieved by adding a purchase threshold constraint to the model of Konno & Yamazaki Simplified by Hamza & Janssen (2000). This amounts to solving a linear mixed program in the case of using the measure of mean-absolute risk.

2.2.1. The Disjunctive Programming and the Problem Modeling

We propose thus a linear disjunctive program as a new modeling of this problem.

Using the linear programming, several criteria can be used for the optimum choice of a portfolio. Young (1998) proposed the norm L_∞ to measure the portfolio risk. It is the "minimax" criterion for the portfolio choice.

Another criterion using the norm L_1 can be used. It is the model of Konno and Yamazaki (1991), which measures the risk through the absolute deviation of portfolio returns compared to its average: $K(x)=E[|R(x)-\mu(x)|]$, where $\mu(x)$ represents the mathematical expectation of the random variable $R(x)$.

Using the historic estimator of $K(x)$[8]:

$$\hat{K}(x)=\frac{1}{T-1}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt}-\hat{r}_j)x_j\right| \qquad \hat{r}_j=\frac{1}{T}\sum_{t=1}^{T}r_{jt}\quad j=1,...,n,$$

And $$\mathrm{y}_t=\left|\sum_{j=1}^{n}(\mathrm{r}_{jt}-\hat{r}_j)x_j\right|$$

[8] Konno H & Yamazaki H. (1991), *A mean Absolute Deviation investment Portfolio optimization Model and its applications to Tokyo stock Market*, Management Science.

The optimization program (P) of the portfolio of Konno and Yamazaki, obtained by the simplified formulation of Hamza and Janssen (1998)[9], is written as follows:

$$Min \quad \frac{\sum_{t=1}^{T} y_t}{T-1} \qquad \textit{subject to}$$

$$\begin{cases} y_t + \sum_{1 \le j \le n} \left(r_{jt} - \hat{r}_j\right) x_j \ge 0 & t = 1,.....,T \\ \sum_{1 \le j \le n} \hat{r}_j x_j \ge \rho & \\ \sum_{1 \le j \le n} x_j = 1 & \\ x_{\mathrm{j}} \ge 0 & 1 \le j \le n \\ y_t \ge 0 & t = 1,.....,T. \end{cases}$$

In the following, the criterion of absolute deviation is retained. For each security

$j\ (j\ =\ 1 \ldots\ n)$, we impose the following condition: $x_j < n_j\ \Rightarrow x_j = 0$, where the constant x_j represents the minimum percentage to invest in the asset $j\ (j\ =\ 1, \ldots, n)$. This is a disjunctive constraint.
So if we introduce the disjunctive constraint which corresponds to a purchase limit in the optimization program, it becomes as follows

$$\text{Min} \ \frac{\sum_{t=1}^{T} y_t}{T-1} \quad \text{Subject to} \quad \begin{cases} y_t + \sum_{1 \le j \le n} \left(r_{jt} - \hat{r}_j\right) x_j \ge 0 & t = 1,...,T \\ \sum_{1 \le j \le n} \hat{r}_j x_j \ge \rho & \\ \sum_{1 \le j \le n} x_j = 1 & \\ x_j < n_j \Rightarrow x_j = 0 & \mathrm{j} = 1,...,n \\ x_{\mathrm{j}} \ge 0 & j = 1,...,\mathrm{n} \\ y_t \ge 0 & t = 1,...,T. \end{cases} \qquad [P_D]$$

Unfortunately, the optimal solution cannot be calculated with conventional variants of the Simplex algorithm for bounded variables.

2.2.2. Diversification and Mixed linear programming

The program of optimization $[P_D]$ is equivalent to the following linear mixed program[10]:

$$Min \ \frac{\sum_{t=1}^{T} y_t}{T-1} \qquad [P_{LM}]$$

[9] Konno H & Yamazaki H (1991), *A mean Absolute Deviation Portofolio optimization Model and its applications to Tokyo stock Market,* Management Science.
[9] Faris HAMZA & Jaqcues Janssen *'Choix Optimal des Actifs Financiers et Gestion de Portefeuille', Hermes-Lavoisier, 2009*

[10] Faris HAMZA & Jaques Janssen *'Choix Optimal des Actifs Financiers et Gestion de Portefeuille', Hermes-Lavoisier, 2009*

$$\begin{cases} y_t + \sum_{1\le j\le n}\left(r_{jt} - \hat{r}_j\right)x_j \ge 0 & t = 1,.....,T \\ \sum_{1\le j\le n}\hat{r}_j x_j \ge \rho & \\ \sum_{1\le j\le n} x_j = 1 & \\ x_j - q_j + u_j = 0 & 1 \le j \le n \\ n_j q_j - x_j + v_j = 0 & 1 \le j \le n \\ q_j = 0 \text{ ou } 1 & \\ x_j \ge 0, u_j \ge 0, v_j \ge 0 & 1 \le j \le n \\ y_t \ge 0 & t = 1,.....,T. \end{cases}$$

Consequently, solving the disjunctive program $[P_D]$ amounts to solving the mixed linear program $[P_{LM}]$.

3. Methodology

3.1. Hypotheses

From the above-mentioned literary review, we adopt the following hypotheses:

Hypothesis 1: The transaction costs sometimes can be very high compared with the expected profitability of the investor, especially for the large portfolios.

Hypothesis 2: the manager of a portfolio cannot anymore fulfill the full diversification, especially when the optimal solutions include very small amounts of investment.

Hypothesis 3: The linear mixed program applied to the mean- absolute-deviation model (Konno & Yamazaki) simplified by Hamza & Janssen ensure the reduction of the size of the portfolio to be optimized and in order to avoid the transaction costs of the financial assets.

3.2. Sample

Our study sample includes data corresponding to the weekly historical returns of 74 financial securities coming from the Casablanca stock market (it makes the totality of the market). The stock market data cover the period of January 2, 2013- June 4, 2014.

For the simulation of the program, an expected rate of return of 0.05% per unit of time (one week) and an accepted minimum threshold of investment of 5% for all j = 1, ..., 74 are planned. Our work is mainly achieved by using *MATLAB* software.

The normality tests are performed on Eviews7.

The distribution of the weekly returns of our portfolio moves far from the distribution of the normal law (see appendix 1).

4. Discussion

The tables below provide us with the optimal compositions of the portfolios obtained by the compared models. The optimal values of the objective function and the time of calculation are referred to in the bottom of the tables.

The first table provides us with the optimal composition of the portfolio obtained by the mean linear model - absolute deviation given by the program (see the theoretical part).

Our optimal portfolio consists of 31 shares distributed according to the different sectors.

Share	Proportion
STOKVIS NORD AFRIQUE	0,16690567
CENTRALE LAITIERE	0,101744341
BMCE BANK	0,085212237
CIMENTS DU MAROC	0,055008352
REBAB COMPANY	0,054966941
TASLIF	0,050693868
DIAC SALAF	0,05038586
HPS	0,046965827
UNIMER	0,040054146
OULMES	0,040049405
DISWAY	0,030701422
AGMA LAHLOU-TAZI	0,028975371
S.M MONETIQUE	0,028220873
ENNAKL	0,023811427
ZELLIDJA S.A	0,022815781
EQDOM	0,021366356
DELATTRE LEVIVIER MAROC	0,021249731
NEXANS MAROC	0,018959653
MAROC LEASING	0,018091
LESIEUR CRISTAL	0,015909345
AFRIC INDUSTRIES SA	0,011971573
REALIS. MECANIQUES	0,01180508
MINIERE TOUISSIT	0,010671625
COLORADO	0,008619764
STROC INDUSTRIE	0,008581833
AFRIQUIA GAZ	0,005485647
FERTIMA	0,00542108
CDM	0,004174023
PROMOPHARM S.A.	0,003929899
INVOLYS	0,003813851
AUTO NEJMA	0,003437894
Objective Function	**0,000265**
Time calculation (min)	**1,32**

Table 1- the portfolio composition aims at a weekly return of 0.05% according to the (P) program.

The second table gives us the optimal composition of the portfolio obtained by the mean linear model - absolute deviation eliminating all the shares, which have a fraction of the order less than 0.0065, from the initial portfolio (that are: AFRIQUIA GAS FERTIMA, CDM, PROMOPHARM **A.C**., INVOLYS, AUTO NEJMA). The optimization leads to an optimal portfolio consisting of 28 securities**.**

In this table related to the constraint that cancels the securities whose rates are below 0.0065, we note that the securities CTM COLORADO, EQDOM and LYDEC, which did not take part in the optimal portfolio resulting from the first optimization program, have come out with lower percentages. So the program is still running and it does not solve the problem.

Share	Proportion
STOKVIS NORD AFRIQUE	0,149337031
CENTRALE LAITIERE	0,092085944
BMCE BANK	0,086006279
REBAB COMPANY	0,073598364
DIAC SALAF	0,055970124
LESIEUR CRISTAL	0,055346167
CIMENTS DU MAROC	0,047779175
OULMES	0,047228567
HPS	0,045244513
S.M MONETIQUE	0,042083581
AFRIC INDUSTRIES SA	0,038411733
TASLIF	0,035913954
AGMA LAHLOU-TAZI	0,028470993
ZELLIDJA S.A	0,028378256
UNIMER	0,026896157
DISWAY	0,023738356
DELATTRE LEVIVIER MAROC	0,023730842
ENNAKL	0,023223734
NEXANS MAROC	0,020157892
MAROC LEASING	0,01388468
MINIERE TOUISSIT	0,010207557
CTM	0,00813781
CARTIER SAADA	0,006965368
STROC INDUSTRIE	0,006174949
MICRODATA	0,005925493
COLORADO	0,002015849
EQDOM	0,001681701
LYDEC	0,001404931
Objective Function	**0,00028**
Time calculation (min)	**1,14**

Table 2- the Composition of the portfolio aims at a weekly return of 0.05% taking into account the constraint according to the (P) program

The results obtained by the optimization program with and without additional constraint are directly compared with those obtained by the linear mixed program in the table below:

Share	Proportion
UNIMER	0,149897
STOKVIS NORD AFRIQUE	0,095441
LESIEUR CRISTAL	0,063806
EQDOM	0,061367
CENTRALE LAITIERE	0,059032
OULMES	0,058744
AGMA LAHLOU-TAZI	0,05698
ENNAKL	0,052154

SALAFIN	0,051414
BMCE BANK	0,05114
TIMAR	0,05012
TASLIF	0,05006
NEXANS MAROC	0,05
RISMA	0,05
CIMENTS DU MAROC	0,05
MAROC LEASING	0,05
Objective Function	**0,001305**
Time calculation (min)	**15,24**

Table 3- the Portfolio Composition aims at a weekly return of 0.05% based on the linear mixed program (P_{LM})

By imposing a minimum threshold of the securities purchase of 5%, the obtained optimal portfolio consists of 16 shares with a rate above or equal to 5%.
To solve this linear mixed problem, we used an algorithm of separation and evaluation (SE)[11] or Branch and Bound (B & B).

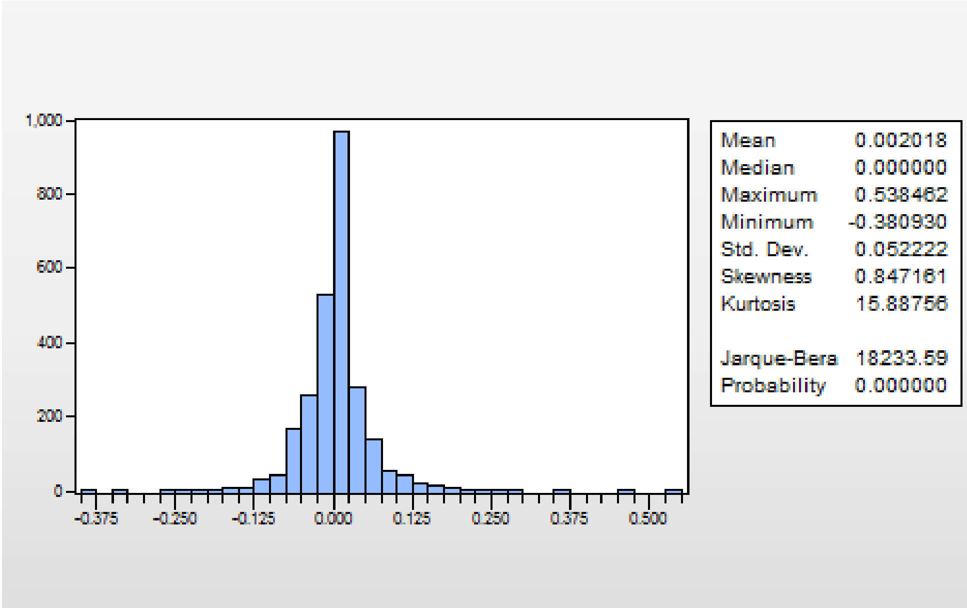
5. Conclusion

The problem of reducing the size of the financial assets portfolio that need to be optimized in order to avoid transaction expenses requires more interest from the researchers. Thus, it has become a major concern for the countries that are aware of the important role of the financial markets in the growth of their economies.
Like most developing countries, Morocco has made radical changes concerning the organization and functioning of the financial markets. The reforms adopted since 1993 have been instrumental in the development of its financial market, but this has not led to a significant improvement in the management of the portfolios of financial assets; particularly the principle of diversification.
Thus, any rational investor within our financial market hopes to own the "right" portfolio; that is to say, a portfolio that offers solely a non-diversifiable risk that he will be paid for. The investor must eliminate the diversifiable risk by optimizing it in any possible way to build a well-diversified portfolio. Nevertheless, he will be left with heavy expenses due to this strategy of diversification.
The advantage of the used method in our stock market is that it ensures an optimal portfolio of a lower number of shares by eliminating all securities with a rate of invested capital less than a minimum percentage of investment.

Appendix

Appendix 1: Graphic 1: Distribution of the weekly returns of the studied sample

[11] Noting that this algorithm is a generic method to solve the problem of optimization, particularly the combinatorial or discreet non-convex optimization. In the methods of separation and evaluation, the separation ensures the obtainment of a generic method to localize all the optimal solutions; whereas, the evaluation avoids the systematic enumeration of all the solutions.



Appendix 2: The code under Matlab according to the $[P_D]$ program

```
functionoptimisation()
[S, txt, tab] = xlsread('price.xls');
[T0,N]= size(S);
for j=1:N,
for t=1:T0-1,
    r(t,j)=(S(t+1,j)-S(t,j))/S(t,j);
end,
end;

disp(r);
[T,n] = size(r);
for i=1:n,
rm(i)= mean(r(:,i));
end;

Y(1:T,1)=1/(T-1);
Y(T+1:T+n,1)=0;
for t=1:T
for j=1:n,
y(t,j) = r(t,j)-rm(j);
end;
end
for i=1:T
for j=1:T
if(i==j)
A(i,j) = 1;
else
A(i,j) =0;
end
```

```
end
for j=1:n
A(i,T+j)=y(i,j);
end
end;
A(T+1,:) =0 ;
for j=1:n
A(T+1,T+j) =rm(j);
end
for j=1:T+n
if(j<=T)
aeq(1,j)=0;
else
aeq(1,j)=1;
end
end
beq=1;

for i=1:T
b(i,1)=0;
end
b(T+1,1) = 0.0005;

Aa=-A;
bb=-b;

m = max(size(Y));

L = zeros(m,1);
[x,fval] = LINPROG(Y,Aa,bb,aeq,beq,L);

for j=1:n
xi(j)=x(j+T);
end
ind=find(xi<=10^-4);
ind2=find(xi>=0.0065);
xi(ind)=0;
xlswrite('Rest_stage1.xls',[txt'  num2cell(xi')]);
ind=find(xi<=0.0065);
ind3=find(xi(ind)>0);
ind2=find(xi>=0.0065);
xlswrite('eli_stage1.xls',[txt(ind(ind3))'  num2cell(xi(ind(ind3))')]);
xlswrite('ind.xls',ind(ind3)+T )
              xlswrite('Value_objective_C_Y1.xls',fval)
```

<u>Appendix 3</u>: The code under Matlab according to the program (P) with an additional constraint

```
function optimisation2()
[S, txt, tab] = xlsread('price.xls');
ind=xlsread('ind.xls');
[T0,N]= size(S);
for j=1:N,
```

```
for t=1:T0-1,
    r(t,j)=(S(t+1,j)-S(t,j))/S(t,j);  %r(i)=ds/s0
end,
end;

disp(r);
[T,n] = size(r);
%rm is the average of return
for i=1:n,
rm(i)= mean(r(:,i));
end;
%disp(rm);
for t=1:T
  Y(t,1)=1/(T-1);
end
for j=T+1:T+n
Y(j,1)=0;
end

for t=1:T
for j=1:n,
y(t,j) = r(t,j)-rm(j);
end;
end

for i=1:T
for j=1:T
if(i==j)
A(i,j) = 1;
else
A(i,j) =0;
end
end
for j=1:n
A(i,T+j)=y(i,j);
end
end;
A(T+1,:) =0 ;
for j=1:n
A(T+1,T+j) =rm(j);
end
for j=1:T+n
if(j<=T)
aeq(1,j)=0;
else
aeq(1,j)=1;
end
end
[k,h]=size(ind);
for i=1:h
aeq(i+1,:)=0;
```

```
aeq(i+1,ind(i))=1;
end
beq(1)=1;
beq(2:h+1)=0;
%IN = eye(n);
%for j=1:n,
% A((T+3+j),:) = IN(j,:);
%end;
%disp(y3);

%U = ones(1,n);
%disp(U);
for i=1:T
b(i,1)=0;
end
b(T+1,1) = 0.0005;
[k,h]=size(A);
%for j=1:n,
%   b(T+3+j,1) = 0.0;
%end;

%disp(b);

%n = max(size(y));
%L = zeros(n,1);
%U = 10^10*ones(n,1);

%disp(size(y));
%disp(size(A));
%disp(size(b));

%X=LP(y,A,b)
Aa=-A;
bb=-b;

% Limites inf?rieures et sup?rieures de x :
m = max(size(Y));
%L = zeros(n,1);
L = zeros(m,1);
%disp(L);
U = ones(m,1);
%
% Optimisation :
[x,fval] = LINPROG(Y,Aa,bb,aeq,beq,L);

for j=1:n
xi(j)=x(j+T);
end
ind=find(xi<=10^-4);
ind2=find(xi>=0.0065);
xi(ind)=0;
```

```
xlswrite('Rest_stage2.xls',[txt'  num2cell(xi')]);
ind=find(xi<=0.0065);
ind3=find(xi(ind)>0);
ind2=find(xi>=0.0065);
xlswrite('eli_stage2.xls',[txt(ind(ind3))'  num2cell(xi(ind(ind3))')]);
xlswrite('ind2.xls',ind(ind3)+T )
xlswrite('Value_objective_C_Y2.xls',fval )
```

Appendix 4: The code under Matlab according to the linear mixed program or Bround & Brunch.

```
Function BB()

%date
[S, txt, tab] = xlsread('cours.xls');
[T0,N]= size(S);
%calculus of return of any share
for j=1:N,
for t=1:T0-1,
r(t,j)=(S(t+1,j)-S(t,j))/S(t,j);
end
end

disp(r);
[T,n] = size(r);
%rm is the of average of return
for i=1:n,
rm(i)= mean(r(:,i));
end;
Y(1:T)=1/(T-1);
Y(T+1:T+4*n)=0;
for t=1:T
for j=1:n,
R(t,j) = r(t,j)-rm(j);
end
end
A(1:T,1:T)=eye(T);
A(1:T,T+1:T+n)=-R;
A(1:T,T+n+1:T+4*n)=0;
A(T+1,1:T)=0;
A(T+1,T+1:T+n)=rm;
A(1+T,T+n+1:T+4*n)=0;

b(1:T)=0;
b(T+1) = 0.0005;
nj=0.05;
Aeq(1:1+2*n,1:T+4*n)=0;
Aeq(1,T+1:T+n)=1;
Aeq(2:1+n,T+1:T+n)=eye(n);
Aeq(2:1+n,T+n+1:T+2*n)=-eye(n);
Aeq(2:1+n,T+2*n+1:T+3*n)=eye(n);
Aeq(2+n:2*n+1,T+1:T+n)=-eye(n);
```

```
Aeq(2+n:2*n+1,T+n+1:T+2*n)=nj*eye(n);
Aeq(2+n:2*n+1,T+3*n+1:T+4*n)=eye(n);
beq(1)=1;
beq(2:2*n+1)=0;
A=-A;
b=-b;
m = max(size(Y));
L = zeros(m,1);

for i=1:n
beq1=beq;
Aeq(1+i,T+n+i)=0;
Aeq(1+i+n,T+n+i)=0;
beq1(1+i)=1;
beq1(1+n+i)=-nj;
[x1,v1,exitflag1] = LINPROG(Y,A,b,Aeq,beq,L);
[x2,v2,exitflag2] = LINPROG(Y,A,b,Aeq,beq1,L);
if(v2<v1)
beq=beq1;
   x=x2;
fval=v2;
else
   x=x1;
fval=v1;
end
end
for j=1:n
xi(j)=x(j+T);
end

for j=1:n
q(j)=x(j+T+n);
end

for j=1:n
xi(j)=x(j+T);
end

for j=1:n
q(j)=x(j+T+n);
end
ind=find(xi>10^-10);
[txt(ind)'  num2cell(xi(ind)')]
sum(xi(ind))
xlswrite('BB.xls',[txt(ind)'  num2cell(xi(ind)')]);
xlswrite('Value_objective_BB.xls',fval )
```

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