The Investigation of the Effect of Patterns of Mathematical Misunderstandings on First Year Engineers' Performance on Mathematical Tasks

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Abstract

This paper presents the results of a project involving fist year university engineering students. Students bring to university a cognitive repertoire of knowledge, including incorrect mathematical knowledge, in the form of patterns of mathematical misunderstandings (=POMM) which may affect both their ability to acquire and apply the new university mathematical knowledge. This effect was investigated by subjecting the students to a POMM test, which was used to determine if there is a correlation between a test of a variety of POMM and students' performance on university mathematical problems.

We found, using Item Response Theory, that that there wasa significant correlation, between a specific latent trait associated with group of engineers' POMM test results, and their main mathematics exam results. At the same time this latent ability depends on the ethnicity of the student.

Key Words: mathematical errors, patterns of mathematical misunderstandings, prior knowledge, mathematical misconceptions, latent traits, ethnicity.

1: Introduction

It is usually taken for granted that underprepared students (low performing mathematics students at university level) lack certain skills, knowledge and know-how. Many first year mathematics students, both prepared and underprepared, bring to the mathematics teaching/learning situation mathematical misconceptions (discussed in 2.1.1 below), incorrect ideas (operations), lack of specific mathematical knowledge, or, as we shall call, *Patterns of Mathematical Misunderstandings* (=*POMM*), relating to the solving of mathematical problems. These *various* POMM are easily identified, yet their causes, and effects, may vary (see Cambell, 1992, Cambell, 2010 and Egodawatte, 2011). Many university mathematics/science lecturers assume, especially in the case of underprepared students, that misconceptions, inherited from students' school mathematics and science education history, has a significant negative affect, on their performance on university tasks (see Hewson & Hewson, 1983; Schoenfeld, 1982 and Solomon, 2006).

In this vein, we decided to investigate the gap, between what students bring from school to university (their *initial, or prior, mathematical cognitive knowledge state*), and what is required of them to perform university mathematics competently, by developing a test of a *variety* of POMM. This was administered to first year engineering students at University (UKZN=University of Kwazulu-Natal) in 2011. These first year engineering students are regarded as "prepared" for University mathematical tasks given their high matric, and maths and science points, used as the main criteria for their admission into the university engineering programme. The significance of a student's POMM on their performance on university mathematics tasks formed the main part of the investigation of this project. The core of the type of errors used to develop the problems in the POMM test will be produced by students at other universities and may differ mainly in those problems where non-mathematical language plays a significant role. By "matric points" we mean the final indicator that a student obtains when leaving school which is used to determine his entrance into university.

2: Theories about POMM and their intervention

2.1 POMM and performance as a guide to the construction of the POMM test

Much research has been done investigating the effect of school students' prior mathematical knowledge, on their performance on university subjects, and/or the development of appropriate intervention material (see for example: Alexander & Judy, 1988; Ball, 1988; Dochy, Segers, & Buehi, 1999; Corbett, Mclaughlin & Scarpinatto, 2000; Duncan & Dick, 2000; Eslinger & Scot, 1996; Halloun & Hestenes, 1985; McIntyre, 2005; Prayag-Beesoondial, 2011; Van de Walle, 2007). Most of these articles involved either considering their *entire* initial mathematical knowledge state, or a *specific* POMM such as problems with mathematics variables, or students' errors in a *specific section of a course*, such as a section of algebra, and not a *variety* of POMM, which our project attempted to investigate. We chose POMM instead of students' entire initial mathematical knowledge they bring to university, given (see Dochy, Segers, & Buehi, 1999): the ease of their accessibility (by identifying them when analyzing students' attempts to solve mathematical problems), and they appeared more amenability to assessment (multiple choice questions = mcq) when testing large classes.

Thus the work reported here involved main research question:
Is there a *correlation* between the POMM results of these first year engineers and their performance on the main mathematics exam in June 2011?

The effect of students' previous knowledge on their performance of new tasks has always been an important aspect of cognitive studies.

Skills students require to solve university mathematics problems

There are no universal explanations or one theory that explains all student mathematical errors (see Egodawatte, 2011).

According to Cambell (1992) there are four main cognitive phases during problem solving:(a)Assimilation,(b) Integration, (c) Planning and (d)Execution Poor previous knowledge may hamper one, or all, of these phases and therefore will affect the outcome of the cognitive task at hand. Identifying these POMM is relatively simple given their repetitive nature and their duration (see Craig & Winter, 1991/1992; Craig & Winter, 1990; Du Toit & Kotze, 2009; Hourigan & O'Donoghue, 2007; Kilpatrick, Swafford & Findell, 2001 and Winter, 1988). These POMM are mostly the result of teaching/learning during their school years (see Djebali, 2004 and Hourigan & O'Donoghue, 2007) that may not adequately prepare learners for the task-demands of university studies (skills, knowledge and competencies) required to solve these successfully and independently.

2.1.1 Classifying types of POMM and the construction of the POMM test

The accumulation of a variety of POMM

The mathematics department at UKZN has collected POMM common to students entering first year mathematics over many years. It was noticed that there were common mistakes that first year mathematical students made both in the class environment and in their exam papers. Over the years these POMMS have been identified (through discussion with other lecturers) and classed according to three different classes, which will be discussed below.

Mathematical misconceptions as a subset of mathematical misunderstandings

Mathematical misunderstandings include students' mathematical misconceptions- when pupils (i) repeatedly use or apply incorrect rules, or (ii) apply correct rules in different areas of application, one has a *misconception* (see Küchemann 1981). To elaborate the type (ii) misconception: when students' conceptions are deemed to be in conflict with the accepted meanings in mathematics, then a mathematical misconception has occurred. Some authors, such as Egodawatte, 2011, exclude type (i) misconception.

There are situations where students apply incorrect schemas (schema is a mechanism in human memory that allows for the storage, synthesis, generalization, and retrieval of similar experiences - see Marshall, 1995) while having the correct ones in their heads. It is natural for a student to try and generalize a given rule – they may do this perfectly correctly or not – hence one must accept that students will make errors and the teacher should try and prevent this before it happens- however this is not always possible. One of the common misconceptions in mathematics arises out of the confusion as a result of the "substitutability" of mathematical variables and the use of the English alphabet in mathematical expressions. In the English language the "g" in the word "fig" can be replaced with the letter "x" creating a *completely different* word. However in mathematics the two very different equations:

$x^2 + 2x + 1 = 0$ and $k^2 + 2k + 1 = 0$

Will have *identical solutions.*

Not all mathematical errors that students commit can be classified as misconceptions.

For example:

(1) $\frac{a}{b+c}=\frac{a}{b}+\frac{a}{c}$

Would be regarded as type (i) misconception, when a student repeatedly makes such an error, and:

(2) $(a+b)^2$

May be a type (ii) misconception of "forcing distribution of power" while:

(3) $-1+\frac{1}{2}=\frac{3}{2}$

May just be a *careless error* and:

(4) $\frac{1}{(8)^{-\frac{1}{3}}}=(-8)^{\frac{1}{3}}=-2$

May be as a result of *lack of knowledge*, or *misreading the problem*and not a misconception, and with problem such as:

(5) Solve: $k^2+2k+1=0$

The student may not be able to proceed at all as there are no "x's" occurring in the equations which he/she is accustomed to.

In the following problem:

(6) If $ab=1$ then:

The student chose the answer: $a=1$ and $b=1$. The student may have read the problem as:

(7) Given $ab=1$, a possible solution is: (the student selects correctly a=1 and b=1).

Also, in the problem

(8) A rectangle is formed in the sea by using a shark net for 3 of its sides and the beach forms the fourth.

The student may not be able to visualize the problem as he/she has never been to the beach and experienced shark nets. In a problem such as:

(9) A window is in the shape of a rectangle surmounted by a equilateral triangle with its base coinciding with one side of the rectangle.

The student may place the triangle inside the rectangle because they have never encountered such a window, or because they have never heard of the word "surmount" or because they confused the dimensionality of the problem (misconception of type (ii)), so the error may, or may not be a mathematical misconception. The student may not be able to proceed to solve the problem at all, as he/she may have never encountered the concept of "shark nets". However a multiple type choice question would force the student to guess in such a situation. Lack of knowledge cannot therefore be regarded as a misconception.

The types of mathematical errors students' commit may affect how the student initially approaches a particular mathematics problem, how the information is processed and integrated, and this will ultimately effect the planning and execution phase. So the error is not necessarily in the calculations, but rather in the initial phases of problem solving. According to Kilpatrick, Swafford & Findell (2001) mathematical proficiency cannot be determined by one term completely; rather it is made up of by five strands, namely: Conceptual Understanding; Procedural Fluency; Strategic Competence; Adaptive Reasoning and Productive Disposition.

We shall adopt these five strands in an attempt to possibly elaborate on **three**, possibility mutual, **categories,** and relate these to the POMM test (Appendix A). This classification is similar to the *Hierarchy of Newman* involving the categories of school children's mathematics errors of 'Comprehension', 'Transformation', 'Process Skills', 'encoding' and 'Carelessness' (see Clements, 1980) but is more aligned with (school) errors identified in University mathematical tasks. In Egodawatte, 2011,the author considered the study of students' errors and misconceptions in four conceptual domains in algebra. These are: variables, algebraic expressions, algebraic equations, and word problems.The author identified 21 errors/misconceptions in Algebra which can be grouped into relevant categories. However there will be errors/misconceptions common to more than one of these areas, and the POMM presented by university students are not all algebraic in nature.

The three categories: Class 1 Type errors, which relate to the RULES/ THEORIES of mathematics, Class 2 Type errors, which relate to the LANGUAGE/ WORD errors associated with interpreting mathematical problems, and Class 3 Type errors, which involve students LACK OF MATHEMATICAL KNOWLEDGE, including misreading the problem or careless mistakes resulting in errors. Class 1 and 3 type errors will be typical errors produced by students at other universities.

Classes of errors and the construction of the POMM test leading to intervention

Problems selected for the POMM test were as a result of university lecturers observing the same "types" of error produced by mathematics students year after year. We attempt to classify the types here and used these categories to assist in the designing of the POMM test. The POMM test was designed to be a multiple choice test given the large number of students involved. We decided on 17 questions to be answered in 1 hour given the constraint that we could only use the students for a double period (90 minutes) before lecturers commenced.

Objective of POMM test

The main objective in the construction of the test was for it to distinguish between a student exhibiting a tendency towards making a POMM, and a student guessing the answers. We therefore gave, as possible choices, the correct answer together with an answer that would indicate a tendency to a mathematical misconception. Given that the POMM test would diagnose those students that committed the most errors and would indicate those areas that students had the most difficulty with, we could then design the appropriate intervention.

We refer to the problems presented in the POMM test (Appendix A) which belong to each category as we discuss them. We also give an example of the answer (included in the choice) that closely reflects the response of a student's misconception of the problem. We found three main classes worthy of investigation, and discuss them in detail and there incorporation into the POMM test:

2.1.1.1 Class 1 type of errors, include errors around the rules that apply to mathematical theory – the "incorrect forcing of a known rule".

For example the correct rule which is already in the student's mind is the Distributive rule over a product. e.g.: $(ab)^2 = a^2b^2$. The student now assimilate a similar problem which they match with this rule incorrectly. For example, asked to simplify the expression: $(a+b)^2$, the student sees distribution in the product, and then follow this rule over a sum, and then states that it equals $a^2 + b^2$, which is incorrect, as it should be $(a+b)(a+b)$. This category will include type (ii) mathematical misconceptions discussed above.

Problems belonging to this category in the POMM test are: Questions: 1, 6, 7, 8, 16, 17.

Question 6 is the question: $(a+b)^2 =$

The possible "force rule" type POMM response would be: $a^2 + b^2$.

2.1.1.2 Class 2 type of errors, include errors around the interpretation of the words, language and concepts occurring in the mathematical problem and involve the interplay of words/concepts used in the real world and formal mathematics.

This would involve the problem solver's ability to interpret the problem correctly, based on their ability to understand the concepts, words and language and translate, or model, this into mathematics.

e.g. A glass window in the shape of a rectangle which is surmounted by an equilateral triangle. Students have to select one of the three figures provided (see question 11 in Appendix A).

In the above example the problem solver has to overcome a few obstacles, which may be mainly of a linguistic nature. The POMM test questions in this category are: Questions: 10,11,13,14. In question 13: "given that 3 pairs of objects can be selected from 3 objects A,B,C, how many pairs of object can be selected from 5 objects A,B,C,D,E?" The answer "5" would reflect a POMM. Students link "3 pairs from 3 objects" with "5 pairs from 5 objects" showing perhaps the effect of the matching words "pairs and objects".

The two POMM categories namely the class 1 and 2 type errors may be a subset of the following POMM:

2.1.1.3 Class 3 type of errors, include errors as a result of *lack* of mathematical knowledge that a student brings to the mathematics problem situation, such as lack of logic or understanding of variables/symbols/mathematical concepts, or misreading the problem or just blatant carelessness.

If an individual POMM cannot be immediately categorized as of class 1 or 2, then it will be slotted into this category. For example the question: "Simplify $\frac{a}{b} = \frac{c}{d}$, $b, d \neq 0$, by forming an equation where there are no variables as denominators".

The resulting error (such as $ab = cd$ or $ac = cd$ instead of $ad = cb$) does not appear immediately to belong to class 1 or 2 type errors above, and may be as a result of the student guessing or using his/her own created incorrect rule, so it is convenient to slot their response into this category. Thus students' *lack* of prior mathematical knowledge may prevent him/her from successfully and independently engaging in university mathematical tasks. For example, the problem: Solve for x: $x^2 = 4$; may get the incorrect response: x=2 and x =-2. This could be as a result of students' lack of logic, including set theory.

Logic is necessary at university – especially for solving mathematical "proof" problems (see Solomon, 2006), where they emphasize the ability to handle proof is a classic example of transfer issues between school and university sectors, and is the focus of a number of well documented complaints regarding students' difficulties in encountering degree-level mathematics .

The POMM test questions in this category are: Questions: 2, 3, 4, 5, 9, 12, and 15. Question 4 states: "if x.0=0 then x= ?". A student exhibiting POMM would more likely select the answer "0" which is a solution- but not the general solution, indicating a possible lack of knowledge involving infinitely many solutions.

2.1.1.4 The base of an individual POMM and its occurrence in a maths problem

Each individual POMM will have an *origin* which may or may not be immediately identifiable.The most common POMM involves, which we would like to term, "forcing of simplification" onto a task (see section 2.1.1.1). For example, students incorrectly "force" the distribution of the power function over addition and subtraction. While $2x+2y$ is equivalent to $2(x+y)$, the following ***expression***may occur in a mathematics problem::

(a) $(x+y)^2$ which is *not* equivalent to: (b) x^2+y^2

The expression (a), the *origin* of the POMM, is referred to as the *base* of the POMM (b).

In this example the base of the POMM is easily identifiable. However, in the problem (see example in 2.1.1.2): "A window in the shape of a rectangle surmounted by an equilateral triangle – sketch this window".

The base of the POMM may be any one (or more) of the words – window, surmounted, equilateral. In this example the POMM base is not immediately obvious – it could also possibly be that there is too much information for the student to cope with, to allow for the proper mathematical interpretation of the story problem.

The base of a POMM occurring in university mathematics problems

University maths courses do not explicitly test individual (school) problems involving just the base of a POMM – however the base of a POMM may occur within a university mathematics problem which may affect the mark that the student obtains for that specific problem.

The two examples below show how a POMM, depending on *where* it appears in the problem, may or may not significantly affect the mark allocation of a problem:

Example 1 – heavy penalizing – POMM base at beginning of problem

The students are required to evaluate the following base problem: $\int\sqrt{1+x^2}\,dx$ (Worth, 10 marks). A student may "force simplify" this base: to get(in three steps):

$$\int(1+x)dx=\int dx+\int xdx=x+\frac{x^2}{2}+c$$

The student would obtain a mark of 0 out of 10 for this attempt, while the actual solution, without this incorrect simplification, will involve at least 10 steps:

This student will thus be heavily penalized for using this POMM and his/her mark significantly negatively affected. However, a different problem involving integrating:

Example 2: little penalty –POMM base at end of problem

Integrate: $\frac{x^{-\frac{1}{2}}}{2}$,between the limits 0 and $a^2 + b^2$ (3 marks) yields the final base answer:

$(a^2 + b^2)^{\frac{1}{2}}$. A student "force simplifying" this result incorrectly, to a + b, will not be heavily penalized (this student will be awarded 2 out of a possible 3 marks) so that the POMM did not have a significantly negative effect on the final mark allocation.

3: Methods

The group which provided the subjects for our research: was the first year intake (n= 650) of students into the mathematics engineering course in 2011. In order to investigate the effect of POMMs on students' performance we have two main objectives: (1) test the first group of students' performance on problems designed around particular mathematical task-demands (=POMM test), (2) to establish the correlation between their performance in this POMM test and their performance on the first year mathematics course in general.

Construction of POMM test: First year engineering students at university have attempted a POMM multiple choice test (see appendix A) involving a collection of past mathematical errors/misconceptions in February 2011 before embarking on their university courses. The POMM questions were developed as a result of analyzing student' performance on first year mathematical problems at university over many years. In the mcq (multiple choice question) test, they were given a problem to solve (e.g. determining the power of two added terms) and asked to select the correct answer: Four choices are given including the correct one and misleading ones involving, for example, the incorrect forced application of a known rule (see different types of POMM in section 2 above). After the questions have been given weighted marks, their percentages obtained in the POMM test are analyzed in relation to their performance on the mathematics 1 paper they wrote in June 2011 (see Djebali, 2004).

In Egodawatte, 2011, students were subject to interviews to determine their errors, as well as their written attempts analyzed and assessed. Given the large number of student is was felt appropriate to use mcq, but this may have negatively effected the validity of the test results.

4: Results and Analysis

4.1Results of tests and presentation of students' data

The results of POMM test (see appendix A) are shown below. The data involving students' answers per question is given below, the correct question has been indicated with an asterisk:

Total number of students = 650
Total number of questions = 17
Number of marked questions = 17
Discrimination sample size = 176
Maximum possible score = 17.0

Average percentage = 57.0
STD deviation in percentage = 14.3
Average Facility Index = 0.57
Average Discrimination Index = 0.35

Question Analysis

Qu	A	B	C	D	E	Omit	Facility	Discrim Index
1	28	23	551*	36	0	10	0.85	0.26
2	15	603*	20	3	0	8	0.93	0.18
3	48	24	32	534*	0	12	0.82	0.28
4	48	67	498*	20	0	15	0.77	0.33
5	39	458	52	79*	1	18	0.12	0.02
6	7	623*	8	1	0	11	0.96	0.11
7	3	12	272	336*	1	25	0.52	0.47
8	4	150	419*	60	0	17	0.64	0.41
9	24	130*	51	431	1	12	0.20	0.14
10	78	466*	58	17	0	20	0.72	0.37
11	181	137	256*	52	0	13	0.39	0.45
12	64	171	211	185*	1	14	0.28	0.32
13	62	360*	91	113	0	21	0.55	0.56
14	70	162	157	237*	0	22	0.36	0.50
15	194*	71	254	113	0	17	0.30	0.30
16	102	90	80	355*	0	18	0.55	0.66
17	471*	17	6	132	0	24	0.72	0.55

Note that the questions with below average discrimination index are questions 1 to 6 and questions 9, 12 and 15. The only questions of this set which match the questions with below average facility index are questions 5, 12 and 15. Question 5 involves error of logic (error type 3), question 12 error of language /dimension (error type 2) and question 15 error of lack of knowledge of plotting graphs (error type 3). These observations are used to development possible intervention material to improve students' performance on mathematical tasks (project in progress).

Reliability of POMM test

There are several forms of reliability measures described in the literature. Nunnally (1972) suggested that three are three possible methods: alternate-form reliability, retest reliability, and split-half reliability. Alternate-form reliability involves two alternate forms of the same test to the same group of students, which did not happen in this case. The retest method, which we did not employ, gives the same test on two occasions. The split-half method needs the same test to be administered on one occasion only, but cannot be used in tests measuring performance.

Validity of POMM test

The validity of a test instrument is just important as itsReliability. If a test does not serve its intended function properly, then it is not valid. According to Remmers (1965), there are four main types of validity: content, concurrent, predictive, and construct. Content validity considershow well the content of the test samples the subject matter. Concurrent validity investigates how well test scores correspond to already accepted measures of performance. Predictive validity deals with how well predictions made from the test are validated by subsequent evidence. This type of validity is not directly relevant to the current study. Construct validity is about what psychological qualities a test deals with. This type of validity is primarily used when the other three types are insufficient. In order to preserve content validity, the content of the POMM test was developed as a result of university mathematics teachers' observing typical errors of students during tutorials, tests, exams and lectures over many years. However, the points allocated to the problems were as a result of a multiple choice test, and whether this is a true reflection of students' POMM status is discussed in the conclusion of this paper

.**Other data**

Students' information pertaining to the items listed below can be obtained from the main author if required: (a) Their results of the mathematics engineering paper in June 2011, (b) Data pertaining to aspects of students such as their Matric points, Matric mathematics and English marks etc.

4.2 Basic statistical methods: analysis of data.

4.2.1 Multi linear regression

The REG procedure in SAS is used to handle multiple linear regressions (MLR). MLR is a method used to model the relationship between a *dependent* variable (also called response variable) and one or more *independent* variables. This relationship can be expressed as an equation that predicts the dependent variable from a linear combination of independent variables and regression parameters. The *regression parameters* are estimated through maximum likelihood iterative procedures such that we obtain an optimized measure of fit, i.e. we want an equation that predicts the dependent variable such that the sum of the squares of the differences between the predicted value and the observed value is minimized. This method is called the method of least squares (see Belsley, Kuh & Welsch, 1980; Draper & Smith, 1981; Hocking, 2003; Hoggs & Ledolter, 1992; Montgomery & Runger, 2010; Myers Montgomery & Vining, 2002; Ott & Longnecker, 2010; Shih & Weisberg,1986; Solomon, 2006 and Yan & Su, 2009 for more detailed stats discussion-detailed statistical analysis can be obtained from the main author if required).

4.2 2 POMM test and the exam

The REG procedure in SAS was applied to 475 students who wrote the POMM test and then the final exam (in 2011) to identify the explanatory variables (including the POMM test) that are significant in determining the MATH1A final mark. The aim of this was to motivate whether the POMM test can possibly be used to identify beforehand whether a student was more likely to succeed in the Math1A final examination or not without any form of intervention.

4.2.3 Is POMM testing able to identify the weaker student?

The REG procedure in SAS was applied to 475 students who wrote the POMM test and then the final exam (in 2011) to identify the explanatory variables (including the POMM test) that are significant in determining the MATH1A final mark. The aim of this was to motivate whether the POMM test can possibly be used to identify beforehand whether a student was more likely to succeed in the Math1A final examination or not without any form of intervention. The independent variables that were considered in the model were the POMM test mark, the race of the student, the gender of the student, whether the student was on financial aid or not, whether the student lived on campus residence or not, whether the student was repeating the module or not and lastly the matric points of the student. The REG procedure in SAS was applied to the data by using stepwise selection. Using the POMM test as a whole, the statistical analysis showed that the POMM test would be a poor indicator of students' performance on first year mathematics, and that school matric points would be a better predictor of students' performance. However using Item Response Theory it will be possible to look at the effect of latent abilities associated with each quesion on student's performance on university mathematical tasks,which we now discuss.

4.2.4 Item Response Theory: latent abilities

Item response theory (IRT) looks at the analysis of true/false or multiple choice questions in a different way compared to just summing the correct responses to determine a student's overall ability in a test. IRT highlights the fact that different questions have different characteristics and two students whose overall scores may be the same might in fact have two different abilities in what the test aims to measure. The most commonly used models are; the Rasch model, also known as the one parameter model, the two parameter model and the three parameter model. IRT is built around the idea that the probability of a correct response for a particular item on the scale depends on the score of a student's latent trait and certain parameters of the scale *(Fischer & Molenaar, 1995).* The three parameters of interest are (i) the difficulty parameter, (ii) the discrimination parameter, and (iii) the guessing parameter. The difficulty parameter is an indication of how difficult an item is and it is the only parameter used in the Rasch model. The discrimination parameter is a measure of how well an item can differentiate between those with weaker abilities in the latent trait and those with stronger abilities in the latent trait. The two parameter model makes use of the difficulty parameter and the discrimination parameter. The guessing parameter measures a student's ability to guess the correct answer (Martin *et. al, 2006).* The three parameter model considers all three parameters in the model. These models are often called the one parameter logistic (1 PL) model, the two parameter logistic (2 PL) model and the three parameter logistic (3 PL) model because the probability of a correct response for a set of latent trait values is modelled using a logistic distribution that also depends on the respective parameters.

Multidimensional models

Figures referred to in this section can be obtained from the author.

A unidimensional 3 parameter logistic (3PL) model was fitted to the data using the "mirt" (*Chalmers, 2012*) package in R 3.0.3 (*R Development Core Team, 2007*). **Error! Reference source not found.Error! Reference source not found.** shows the item characteristics by plotting the item trace lines for each question. The plot shows the probability of a correct response for a range of underlying latent trait scores. We can see that some of the items have somewhat flat discrimination (questions 6, 9 and 15). Question 5, on the other hand, reveals a reverse discrimination. This means that people with lower values of this trait had a higher probability of answering the question correctly. This could possibly infer that this latent trait is representative of an inability score rather than an ability score. However to fit a unidimensional model, the latent trait must discriminate all items in the same direction or else it makes no sense when interpreting the scale as a whole.

The item fit statistics show that there are items that do not fit the model well. Further analyses is performed with the packages "mokken" (*van der Ark, 2007)* and "KernSmoothIRT" (*Maza et. al., 2014*) to investigate whether there are violations in the assumptions of monotone homogeneity, local independence or unidimensionality. Table 1 shows the output of these tests.

When the scalability of the items are investigated, we see that questions 5 and 9 have negative item coefficients and other items have coefficients very close to zero. This means that the assumptions of unidimensionality or local independence are most likely not satisfied for the scale. Table 1 also shows negative coefficient signs for questions 5 and 9 when we investigate the monotonicity in the items, showing clear violations in the assumption. Other items have coefficients close to zero too. The monotonicity assumption can be checked graphically in **Error! Reference source not found.** and **Error! Reference source not found.** where the violations can clearly be seen for some items and potentially seen for others.

Overall we can see that there are violations of monotonicity in the item characteristics, local independence or unidimensionality amongst more than a few of these items. Using the "mokken" package we can explore the partition of these items into scales. In doing so, the maximum number of suggested scales is three.

We will however proceed with fitting a two factor multidimensional 3PL model. Once the model is fitted, we compare the fit of the unidimensional and two factor multidimensional models using the likelihood ratio test. We see that the two factor model fits the data significantly (p-value < 0.001) better than the unidimensional model, since the corrected AIC (Akaike's Information Criterion) value is higher for the unidimensional model. The AIC value measures the relative fit of a model and is only meaningful when comparing the fit between two or more models (the lower, the better). **Error! Reference source not found.** and **Error! Reference source not found.** show the trace surfaces for the two factor multidimensional models specifically for question 5 and question 13 respectively. θ_1 and $\theta_2$respectively represent the two distinct latent traits and $P(\theta)$ represents the probability of getting a correct response. The trace surfaces for all questions were investigated but questions 5 and 13 were selected for this discussion because in the unidimensional model, question 5 fitted poorly while question 13 provided a very good fit to the model. Now looking at **Error! Reference source not found.** we see that the response for question 5 shows a very strong dependency on the second latent trait and very little (almost none) on the first latent trait. Looking at **Error! Reference source not found.** we see firstly that the shape of the surface for question 13 highlights potential violations for the estimation in the ability estimates. Questions 12 and 14 shows similar surface shapes. The surface shapes for the rest of the questions show that the second latent variable has little to no effect on the probability of a correct response.

As suggested by the "mokken" package, we will fit a three factor multidimensional 3PL model. Once the model is fitted, we compare the fit of the two factor multidimensional model and the three factor multidimensional model using the likelihood ratio test. We see that the three factor model fits the data significantly (p-value < 0.001) better than the two factor model, since the corrected AIC value is lower for the three factor multidimensional model. Since the orientation of the factor loadings is arbitrary the initially extracted solution should be rotated to a simpler structure to better facilitate interpretation. An oblimin rotated factor solution, suppressing absolute loadings less than 0.2, was found. A more restricted confirmatory model was then built using the factor loadings generated by the oblimin rotated factor solution for the three factor multidimensional 3PL model. There is no significant difference (p=0.1662) between these two models but the restricted confirmatory model will be used to estimate the latent traits because it had a lower corrected AIC value, indicating the possibility of a better fit.

According to the model there are three latent traits that can explain the results for the scale.

Table 2 shows the factor loadings of each question onto the different latent traits. The stronger in magnitude these factor loadings are, the stronger the influence of that particular question on that particular trait. This can be used as a guide to identify what each latent trait could most likely represent.

The latent traits/abilities were then estimated for each subject by using the final model. The next step is to investigate the effect of these latent traits on the final exam mark using a multiple linear regression model in STATA13 (*StataCorp, 2013*).

The exploratory analyses first considered determining whether the effects of the latent traits on the exam marks were significant or not. The second latent trait was found to have a significant effect on the exam mark such that a one unit increase in the second latent trait implies a 9.5% increase in exam mark ($p < 0.0001$). When latent trait one and three are removed from this model, the effect decreases slightly to 8% but it is still statistically significant ($p < 0.0001$). This latent trait may involve the POMM where carelessness is involved- i.e. the trait may belong to class 3 of POMM.

Since it is known that education differences are almost always associated with ethnicity, the role of ethnicity was investigated. Black, White and Indian student groups were considered since these are the three largest ethnic groups in KwaZulu-Natal and make up the three largest student populations at UKZN. The average exam marks for the Black, White and Indian student groups were 51.79%, 46.65% and 50.37% respectively and an analysis of variance test found no significant differences between these averages ($p = 0.3192$).

However, when ethnicity is added to the regression model with the second latent trait, the significance of ethnicity becomes detectable. The effect of the second latent trait can be interpreted as; adjusting for race, a one unit increase in the second latent trait implies an average increase of 8.6% in exam mark ($p < 0.001$). The effect for race can also be seen as; when holding the second latent ability constant, White students score on average 8.8% less than Black students ($p = 0.007$) and Indian students score on average 4.5% less than Black students ($p = 0.014$).

Since ethnicity is in the way of the causal pathway between the second latent trait and the student's exam mark, we should also investigate the effect of ethnicity on the second latent trait. Looking at a linear regression model, a significant effect of ethnicity on the second latent trait can be seen. A White student will, on average, have a second latent trait score of 0.45 more than a Black student ($p = 0.001$). On average, an Indian student will have a second latent trait score of 0.39 more than a Black student ($p < 0.001$). These latent trait scores aren't really meaningful. However, what we can see from the constant term (-0.23) in this regression is very telling. Since we know that a score of 0 means an average ability in the latent trait, this means that the average Black student has a lower than average ability in this latent trait, while the average Indian and White student has a higher than average ability in this latent trait. If we regard this trait as "carelessness" it appears that Black students are less careless when dealing with problems such as question 5 in the POMM test.

So it is interesting to see that this latent ability that is being measured by the POMMS scale has a direct effect on the exam mark – but at the same time, this ability depends on the ethnicity of the student. It is also interesting to note that if all students could be set to the same level for this latent trait then previously disadvantaged students would be performing better.

5: Discussion

5.1 POMM and the exam

5.1.1 POMM as a poor predictor of university mathematics performance

The statistical analysis above gave a clear indication that only considering the POMM test mark would be a very weak indicator of mathematics students' performance on university mathematical tasks if it were to be used alone. Using the POMM test mark in combination with all the other significant variables, the information that POMM test attributes would be negligible. The matric points of a student was a good indicator for their performance on the maths exam. However considering students' latent ability associated with the attempts to solve the problems in the POMM test revealed a significant correlation with the main mathematics university exam.

6: Conclusions

6.1 POMM and performance

Matric points is of course a PROXY variable for what someone has learned to know and do after matric, including their prior mathematical knowledge they bring to university, and as such, is an index of their prior learning, and to the degree that the POMM test final mark did not predict as well as matric points, our findings indicate that the POMM test we devised may have been mistargeted.

The POMM test designed for this project may have been not a proper reflection of the many errors students bring to the mathematics teaching and learning situation. A better POMM test may be designed to cover more material in more depth which may increase the proportion of variation in the mathematics exam. The POMM test should incorporate more difficult problems and even unseen ones. It may be necessary for the students to be made aware of the importance of the POMM test – e.g. that the POMMS test counts towards the exam, so that they will take it more seriously.

Using multiple choice questions is always very dangerous in terms of accurate prediction of students' performance. It is impossible to check the students method as one is only presented with their final answer – they may have understood a specific problem fully and used a mathematically sound method to solve it, but the final interpretation may have been off which meant they were not rewarded anything for their attempt. In a written problem situation they would have been awarded marks and not been given zero for their attempt.

There is also the effect of random selection of answers in a mcq test on students' performance which could effect any statistical analysis of the data collected.

However when we used Item Response Theory we found that a specific latent ability that is being measured by the POMMS scale has a direct effect on the exam mark – but at the same time, this ability depends on the ethnicity of the student.

With respect to the student's performance on the final exam, the occurrence of the base of a POMM within a problem in terms of when it is encountered can have a significant effect of the students final mark for the problem – the earlier on the base is encountered the more likely the student will get very little marks for their attempt.

6.3 Conclusion - POMM test

The statistical analysis showed, when using the POMM test mark, that there was no significant correlation between the POMM test results and the students' performance on the UKZN mathematics exam. However Item Response Theory showed that students' latent ability realized by the POMM test had a significant effect on the main exam – students which were "less careless" appeared to perform better on the main exam, and that this latent ability is also affected by ethnicity.

The question in the POMM test which gave the most information about the effect of students' latent ability was question 5- this involved "If …., then…." which contains English words not familiar to non-English speaking students – hence ethnicity would play a part in performance on such problems.

The fact that there were a large number of students to be tested, mcq type POMM test seemed to be the obvious choice. However, not all questions, involving POMM involved just a true or false possibility, so that students' strategies, or lack thereof, may not have been identified.

In Huntley, Engelbrech & Harding (2009) the reliability of multi-choice testing in assessing mathematics students at first-year tertiary level has always raised controversy. Since 1972, such tests have been incorporated into the major mathematics subject, Mathematics 1A, at the South Australian Institute of Technology. A detailed analysis of the correlation between the multi-choice testing and other forms of assessment was made after 1973. The results were surprisingly extremely well correlated (statistically speaking) but, nevertheless, there were still a few glaring discrepancies which must cast doubt over the consistency of different forms of assessment.

In Wainer & Thissin (1993) MCQ is regarded as economically practical and allows for reliable, objective scoring but may focus on student recall rather than generation of answers. However in Colgan (1977) it is emphasized that constructive-response is more difficult to score objectively, but they provide a task that may have more systemic validity. When these two types of testing (mcq and constructive-response) where used in examining students' performance on computer-science tasks, they found no evidence that the two components of the test measure different aspects of proficiency – but this may be different with testing students' performance on mathematical tasks. They show, in terms of weighting, it is not practical to equalize the reliability of the different components.

The research in Williams (2006) reflects the ongoing debate surrounding the usefulness (or otherwise) of mcq as an assessment instrument and the need for a more "reliable" form of mcq. The use of assertion-reason questions (arq), an apparently sophisticated form of mcq that claimed to encourage higher-order thinking on the part of the student, was investigated. However their research cast doubt over whether student performance in arq tests can, indeed, be looked upon as a good indicator of deeper learning—student reactions and opinions suggesting instead that performance might have more to do with one's linguistic competency.

The number of POMM and type of POMM tested may not have been sufficient to provide an accurate assessment of their POMM status so future POMM tests may need to be expanded, perhaps inserting more problems similar to question 5 of the POMM test.

6.4 Hypothesis- true or false?

In this research article we posed a hypothesis involving the effect of school mathematics learning on first year students' on performance on university mathematical tasks. Mathematics teachers take for granted that students with a high number of mathematical misunderstandings will perform poorly on first year university tasks. We found, surprisingly, that when considering the POMM test marks, that there was no correlation between students' variety of patterns of mathematical misunderstanding developed from their schooling. The fact that there was a correlation between a specific latent abilityreveal by a specific problem in the POMM test and students' performance on university mathematical task will provide guidance for future construction of POMM tests.

Table 1: Scalability and monotonicity check coefficients

Item	Scalability coefficient	Monotonicity coefficient
Question 1	0.076	0.08
Question 2	0.208	0.21
Question 3	0.054	0.05
Question 4	0.088	0.09
Question 5	-0.119	-0.12
Question 6	0.058	0.06
Question 7	0.066	0.07
Question 8	0.057	0.06
Question 9	-0.010	-0.01
Question 10	0.057	0.06
Question 11	0.073	0.07
Question 12	0.081	0.08
Question 13	0.120	0.12
Question 14	0.091	0.09
Question 15	0.026	0.03
Question 16	0.177	0.18
Question 17	0.145	0.15

Table 2: Factor loadings of each question onto the different latent traits

Item	Latent Trait 1	Latent Trait 2	Latent Trait 3
Question 1	0.428		
Question 2	0.949	-0.267	
Question 3	0.492	0.217	
Question 4	0.261	0.493	
Question 5	0.277	-0.984	
Question 6		0.897	-0.330
Question 7	0.612		
Question 8	0.986		
Question 9	-0.238		
Question 10	0.594	0.704	
Question 11		0.232	
Question 12			0.378
Question 13		0.212	0.367
Question 14			0.326
Question 15	-0.369		0.407
Question 16	0.422		0.842
Question 17			1.000

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APPENDIX A: RESEARCH TEST MATHS 1 ENGINEERING 2011

Your results will be used for research purposes only. Your assessment of your performance will not be used to disadvantage you in any way.

INSTRUCTIONS:

(1) PUT YOUR NAME **AND** STUDENT NUMBER ON THE MUTLIPLE CHOICE QUESTIONAIRE (M.C.Q).

(2) CIRCLE YOUR CHOICE OF ANSWER TO EACH QUESTION ON THE M.C.Q IGNORING THE T (TRUE) F(FALSE OPTION) (a,b,c in the questions are real numbers).

(3) HAND IN THIS QUESTION PAPER **TOGETHER** WITH YOUR M.C.Q.

Question 1

If $f(x) = -x^2 + x - 1$, then $f(x+1) =$

(a) $-x^2 + x - 1 + 1 = -x^2 + x$ (b) $-(x+1)^2 + (x+1) - 1 + 1$ (c) $-(x+1)^2 + (x+1) - 1$

(d) Neither of a,b,c.

Question 2

The equation of a straight line is $y = mx + c$, where m is the slope of the line. The straight line $2y - x = 4$ has slope:

(a) 2 (b) 1/2 (c)-1/2 (d) Neither of a,b,c.

Question 3

The parabola $y = ax^2 + bx + c, a \neq 0$ has axis of symmetry $x = -b/2a$.

The parabola $2y - 4x^2 + 8x = 3$ has axis of symmetry:

(a) 2 (b) 1/2 (c) -2 (d) Neither a,b,c.

Question 4

If $x.0 = 0$ then $x =$

(a) 0 (b) undefined (c) any real number (d) Neither a,b,c.

Question 5

If $a.b = 1$ then

(a) $a. = 1$ or $.b = 1$ (b) $a. = 1$ and $.b = 1$ (c) $a. = 2$ or $.b = 1/2$ (d) Neither a,b,c.

Question 6

$(a+b)^2 =$

(a) $a^2 + b^2$ (b) $a^2 + 2ab + b^2$ (c) $a^2 + ab + b^2$ (d) Neither a,b,c.

Question 7

$\sqrt{a^2 + b^2} =$

(a) $a^2 + ab + b^2$ (b) $a + 2ab + b$ (c) $a + b$ (d) Neither a,b,c.

Question 8

$\frac{a}{b+c} =$

(a) $\frac{b+c}{a}$ (b) $\frac{a}{b} + \frac{a}{c}$ (c) cannot be simplified (d) Neither a,b,c.

Question 9

If $\frac{a}{b} = \frac{c}{d}$, $b, d \neq 0$, then

(a) $ac = bd$ (b) $ad = cd$ (c) $ab = cd$ (d) Neither a,b,c.

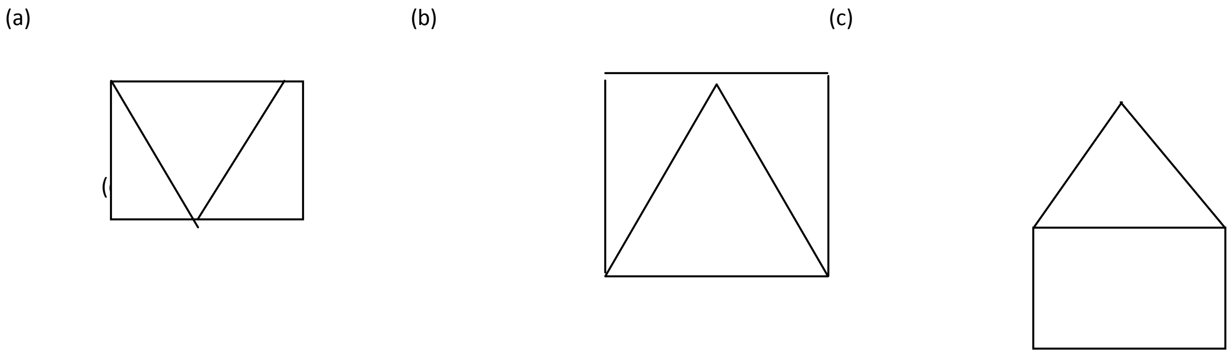
Question 10

Two men can dig a hole of fixed volume V each at the same rate p cubic meters per second. It takes them one hour to dig this hole. One man, digging the same hole at the same rate p, will dig the hole in:

(a) One hour (b) two hours (c) 1/2 hour (d) neither a,b or c

Question 11

A window is in the shape of a rectangle capped/surmounted by an equilateral triangle, the base of the triangle coinciding with one of the sides of the rectangle. The diagram for the window is:



Question 12

If the length of the sides of the triangle in question 11 is x, and the length of the remaining side of the rectangle is y, then the perimeter of the window is:

(a) $2y+5x$ (b) $2y+3x$ (c) $2y+4x$ (d) Neither a,b,c.

Question 13

The pairs which you can select from three objects A,B,C are

AB; AC and BC; i.e. 3 pairs can be selected from three objects. So the number of pairs that can be selected from 5 objects is:

(a) 5 (b) 10 (c) 20 (d) Neither a,b,c.

Question 14

If 4 horses P,Q,R,S take part in a race, then the possible outcomes of the positions first, second (no ties) in the race are:

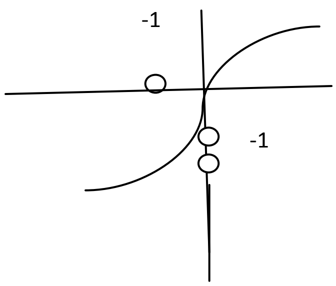
P(first), Q(second) or Q(first), P(second) or P,R or R,P or P,S or S,P or Q,R or R,Q or Q,S or S,Q or R,S or S,R which yields 12 possible outcomes. The number of possible outcomes for position first, second and third (no ties) involving the 4 horses will be:

(a) 4 (b) 20 (c) 16 (d) Neither a,b,c

Question 15

The graph of y=f(x) is given below. Using the graph, the approximate value of f(-1) is:

(a) -1 (b) 0 (c) -2 Neither a,b,c.



Question 16

$\frac{\sin(3x)}{3} =$

(a) $\frac{\sin 3.\sin x}{3}$ (b) $\sin x$ (c) $\sin 1.\sin x$ (d) Neither a,b,c.

Question 17

$\cos(a+b) =$

(a) $\cos a \cos b - \sin a \sin b$ (b) $\cos a + \cos b$ (c) $\sin(a-b)$ (d) Neither a,b,c.

Question 18

My matric mathematics symbol is:

(a) A (b) B (c) C (d) Neither a,b,c