NUMERICAL SIMULATION OF TURBULENT NATURAL CONVECTION IN A RECTANGULAR ENCLOSURE WITH LOCALISED HEATING AND COOLING

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Abstract

This study involves simulation of turbulent natural convection in a rectangular enclosure with localised heating and cooling. Numerical simulation of turbulent natural convection has been studied in the past using the k-epsilon (k-ε), k-omega (k-ω) and k-ω-SST turbulence models. Further research showed that the k-ω SST model performed better than the k-ε and k-ω models. The study of natural convections in an enclosure has several applications from natural space, warming of household rooms to sections of engineering and atomic installations. This study involves numerical simulation of natural convention flow in a rectangular enclosure full of air using the k-ω- SST model with an objective of establishing the best position of the heater and the cooler for better distribution of heat in the enclosure. The transfer of heat due to natural convection inside a rectangular closed cavity was modelled to include the effect of Rayleigh number greater than or equal to 10^9.

The non-linear terms in averaged momentum and energy equation respectively were modeled using k-ω-SST model to close the governing equations.

The cavity was maintained at 303K on a square hot section midway on the extreme lower boundary of one of the vertical walls and at 283K on a square (twice in length and width the lower one) cold section midway on the extreme upper boundary on the same wall. The remaining part of this wall and the other five walls were adiabatic. The vorticity-vector potential, energy and the two equations for k-ω-SST model with boundary conditions were solved using finite difference method and FLUENT.

Key words: Simulation, natural convection, localized heating and cooling,Rayleigh number.

1. Introduction

Natural convection is a mechanism or type of heat transport in which the motion of the fluid is not generated by an external source but only by the difference in densities in the fluid occurring due to temperature gradients.

Turbulent flow exists everywhere in nature from the jet stream to the Oceanic currents. Turbulent flows tend to occur at higher characteristic linear dimension. If Re>3500, then the flow is turbulent. They are characterized by the regular and disorderly movement of the particles of the fluid.

Turbulent flows are highly irregular and random, have high diffusivity and are described by a strong 3-D vortex generation mechanism called vortex stretching.

The study of natural convections in an enclosure has several applications from natural space, warming of household rooms to sections of engineering and atomic installations such as material processing, cooling of electronic equipment building technology as well as in passive heat removal system of a liquid metal nuclear rights actor characterised by four main features, dimension, dissipation, three dimensionality and length scales.

2. Governing equations

The representing equations in two dimensional rectangular directions;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=F_X-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=F_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

$$\rho C_P\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\phi \quad (4)$$

Where

$$\phi=\mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right\}$$

3. Dimensionless Energy, Momentum and Continuity Equations

Non-dimensionalizing involves the partial or full removal of units from an equation involving physical quantities by a suitable substitution of variables. This makes the equations simpler and highlights which terms are the most important. The main objective behind non-dimensionalization is to lessen number of variables.

The set of equation in dimensionless form becomes:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (5)$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (6)$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ra.Pr.\theta_f \quad (7)$$

$$\left(\frac{\partial \theta_f}{\partial \tau} + U\frac{\partial \theta_f}{\partial X} + V\frac{\partial \theta_f}{\partial Y}\right) = k\left(\frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2}\right) + \phi \quad (8)$$

Where, Pr and Ra denotes Prantdl and Rayleigh numbers correspondingly; and θ_f is the dimensionless fluid temperature.

Prantdl number, $Pr = \frac{V}{\alpha^2}$

Rayleigh number, $Ra_L = Gr_l Pr = \frac{g\beta(T_S - T_\infty)L^3}{v^\propto}$

4. Mathematical Formulation

In this study numerical simulation of turbulent natural convection within a rectangular enclosure is studied. The physical situation is illustrated by the fig below,

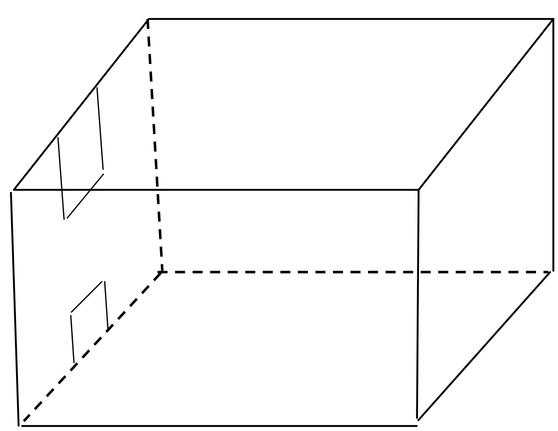


Fig. 1 Geometry of the problem

The cavity was maintained at 303K on a square hot section midway on the extreme lower boundary of one of the vertical walls and 283K on a square (twice in length and width the lower one) cold section midway on the extreme upper boundary on the same wall. The remaining part of this wall and the other five walls are adiabatic.

The fluid is initially motionless and at a uniform temperature equal to the average temperature of the vertical walls. The temperature of the hot sectionT_h and that of the cold section $T_c$is constant with$T_h > T_c$. Under this condition the density gradient of the internal fluid is normal to the gravity and the buoyancy –driven natural convection starts immediately when the heat is applied.

Due to buoyancy, a fluid motion is enclosed in the enclosure depending on the enclosure geometry (aspect ratio), the working fluid and the temperature difference .In this study, L=1m, W=$\frac{1}{2}$ and D=0.32.The working fluid is air. Fluid flow therefore depends only on the temperature difference $dT= T_h > T_c$.In terms of dimensionless analysis, the presentative parameter is the geometrical aspect ratio. The aspect ratio is $A_x=\frac{l}{w} = \frac{2}{1}$=2, $P_{r=} \frac{v}{\alpha}$=0.72 and Ra =$\frac{g\beta\rho^2 c_p dt\, L^2}{\mu k}$

The temperature difference is chosen to give the Rayleigh number of interest. For this study Rayleigh number is varied from $10^{10}$to 10^{13}.

The problem is to find the subsequent velocities and temperature as a function of time and position and the rate of heat transfer across an enclosure as a function of time. The third dimension is large enough so that the flow and heat transfer are two dimensional.

5. Methods of solution

The finite volume based solver fluent 6.3.26 with boussinesq approximation is used to solve the above governing equations.

6. Results and discussion

The results presented here were obtained resolving the governing equations mathematically by utilizing finite difference technique together with boundary situations given the numerical solutions for variables in $k - \omega - SST$ model. In this study the results are obtained for different Rayleigh numbers i.e. $10^{10}, 10^{11}, 10^{12}, 10^{13}$ and results of isotherms streamlines and contours of velocity magnitude are recorded at$z = 0.5$.

7. Isotherms

Isotherm is a line of equal or constant temperature or is a curve on a graph that connects points of equal temperature. In the fig 6.11 the maximum temperature is 257K, in fig 6.1.2, the highest temperature is 249K, in 6.1.3, the highest temperature is 230K, and in 6.1.4, the highest temperature is 199K.

The high temperature are evident on the left side wall. In all cases two round motion in opposite directions (clockwise and anticlockwise direction). There is rising up of hot less dense particles which loses its heat with distance as shown by change in color. In between the two isotherm walls, there is mixing of air particles which is a relatively warm region. In 16 c and d temperature uniformity is achieved. In conclusion it is evident that maximum temperature decreases with increase in Rayleigh number.

Fig 6.1.1 Isotherms of Rayleigh number $\mathbf{10^{10}}$**.**

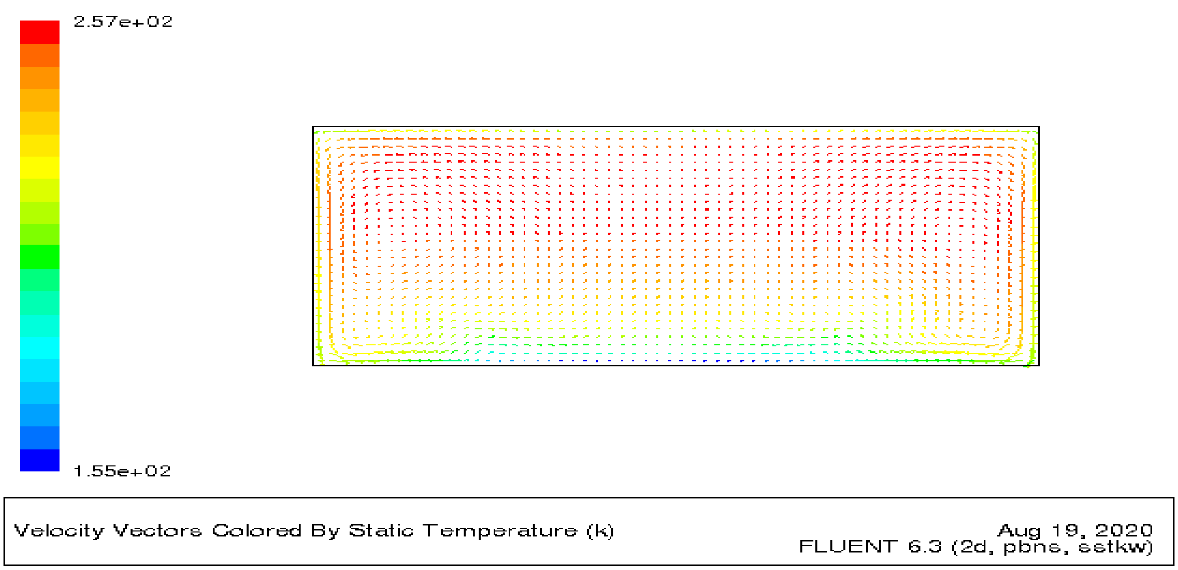


Fig 6.1.2 Isotherms of Rayleigh number $\mathbf{10^{11}}$.

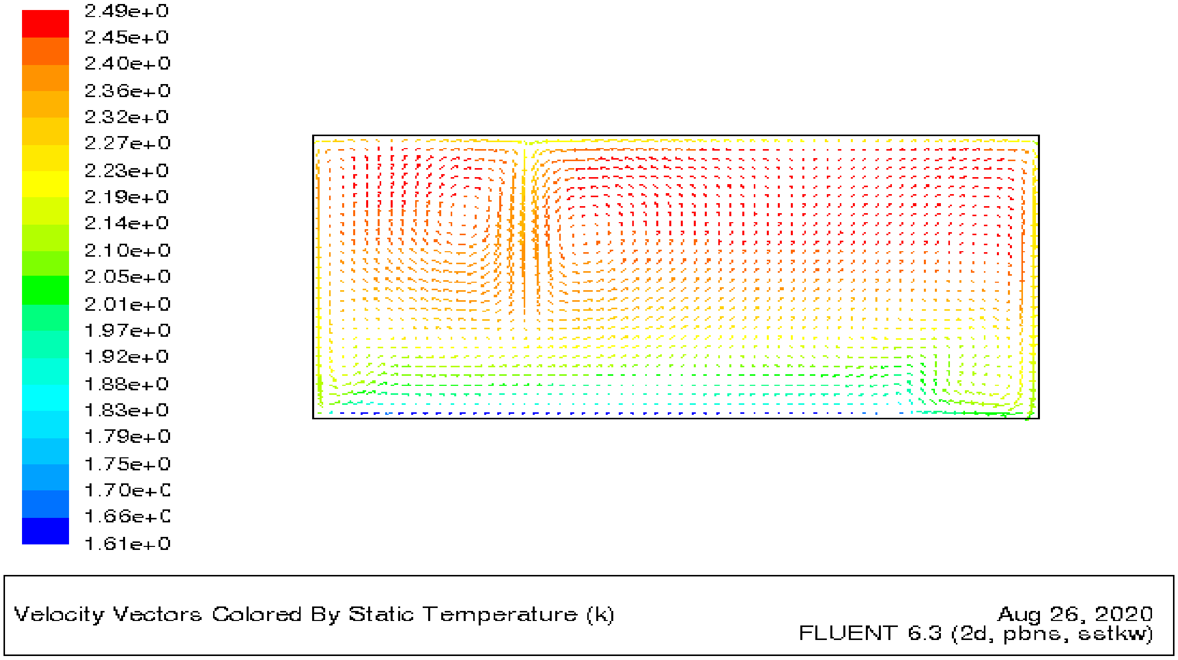


Fig 6.1.3 Isotherms of Rayleigh number $\mathbf{10^{12}}$.

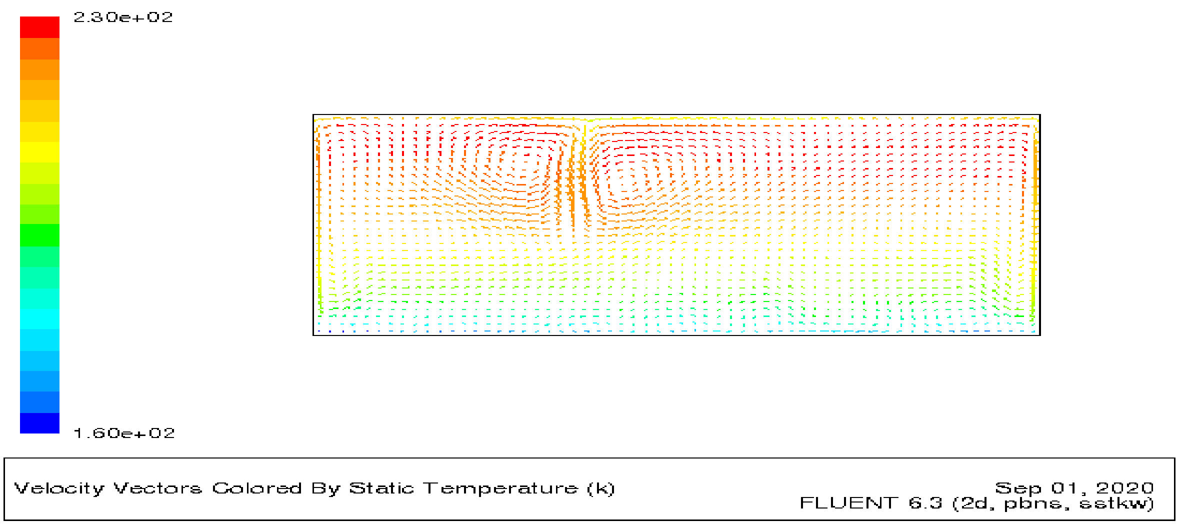
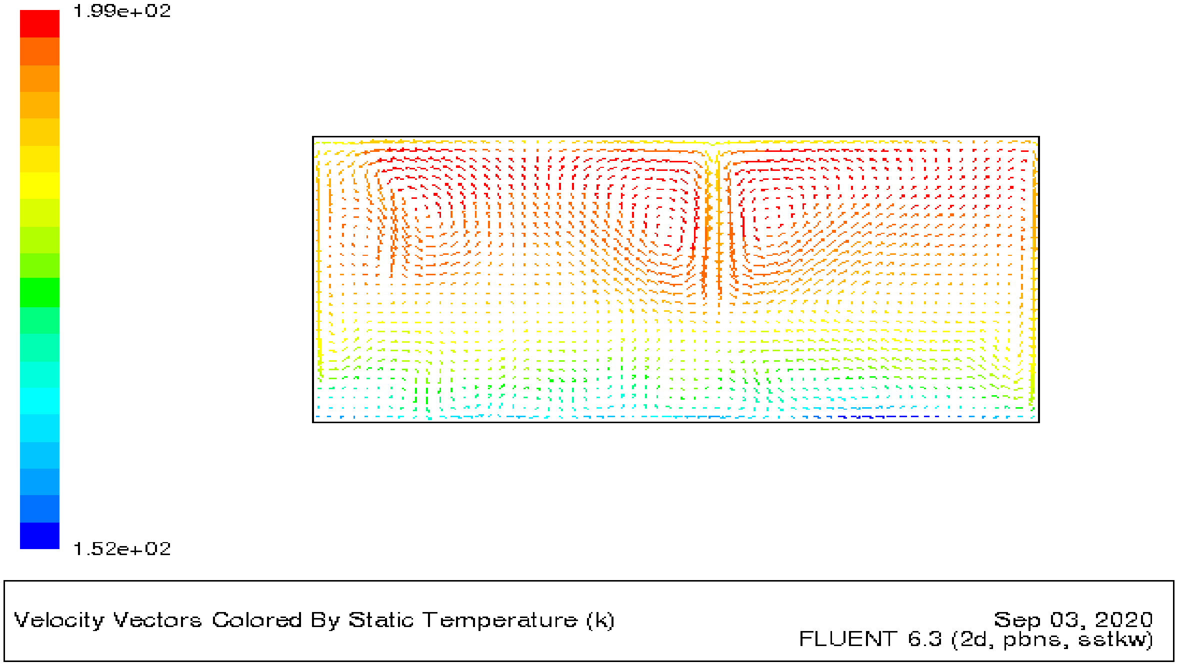


Fig 6.1.4 Isotherms of Rayleigh number $\mathbf{10^{13}}$.



6.2 Contours of velocity magnitude

In 6.2.1, the highest velocity of air particles is 0.522m/s, in 6.2.2, the highest velocity is 0.555m/s, in 6.2.3, the highest velocity is 0.593m/s and in 6.2.4, the highest velocity is 0.870m/s. In 6.2.1 the highest speed is at the middle at the mixing region. Vortices are more in 6.2.1 which become parallel as the Rayleigh number increases in 6.2.4 the vortices are parallel than any other set-up in this study and at this point is evident that as the Rayleigh number increases the flow becomes less turbulent.

Fig 6.2.1 contours of velocity magnitude of Rayleigh number $\mathbf{10^{10}}$.

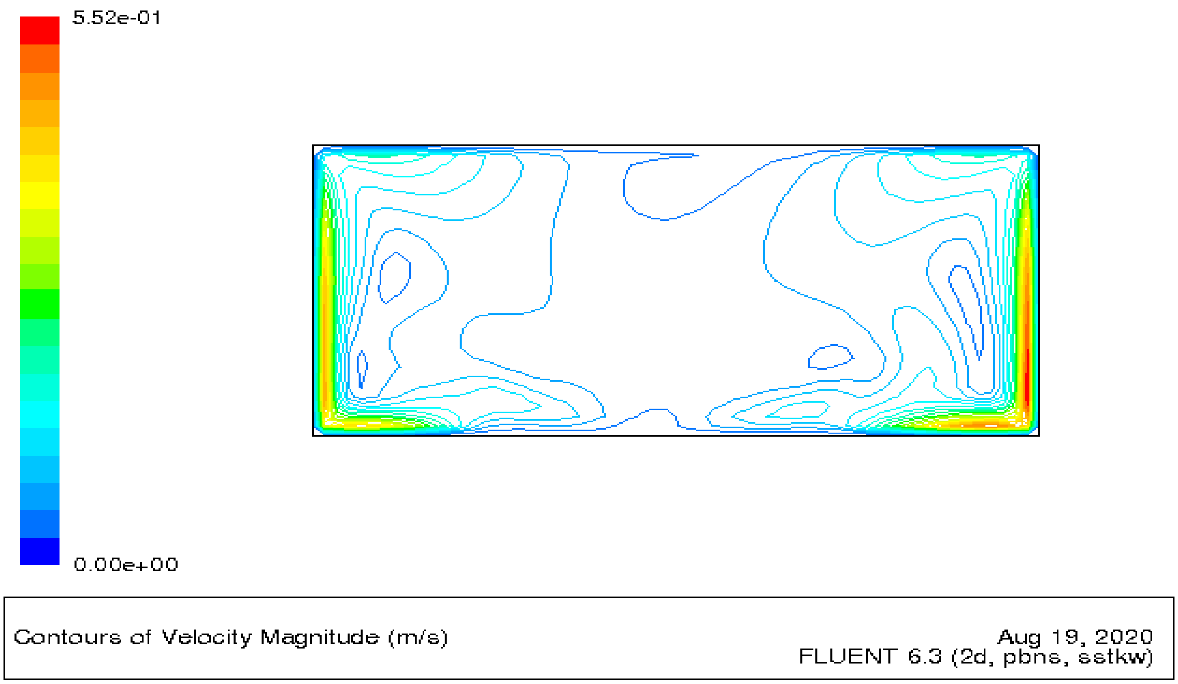


Fig 6.2.2 contours of velocity magnitude of Rayleigh number $\mathbf{10^{11}}$**.**

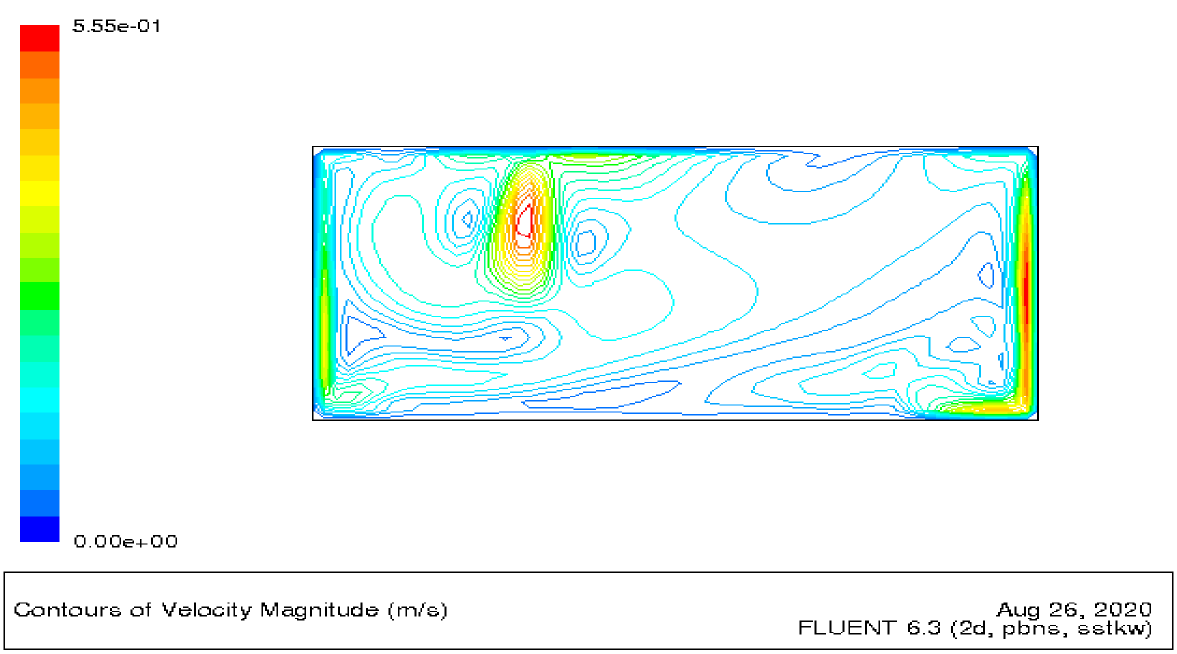


Fig 6.2.3 contours of velocity magnitude of Rayleigh number $\mathbf{10^{12}}$.

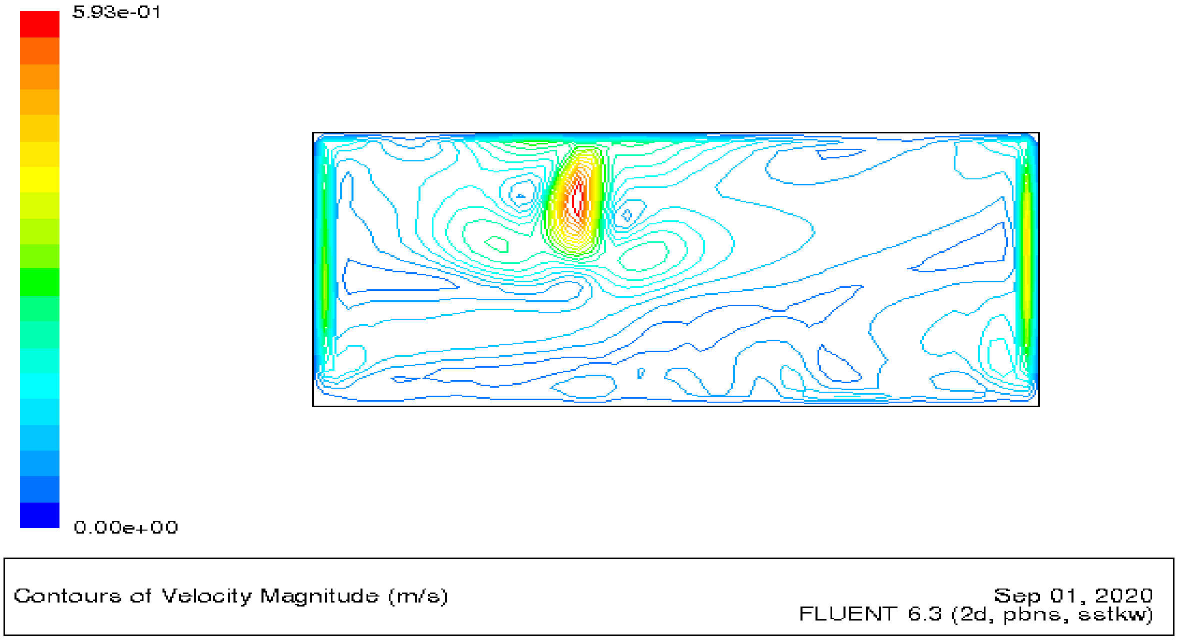
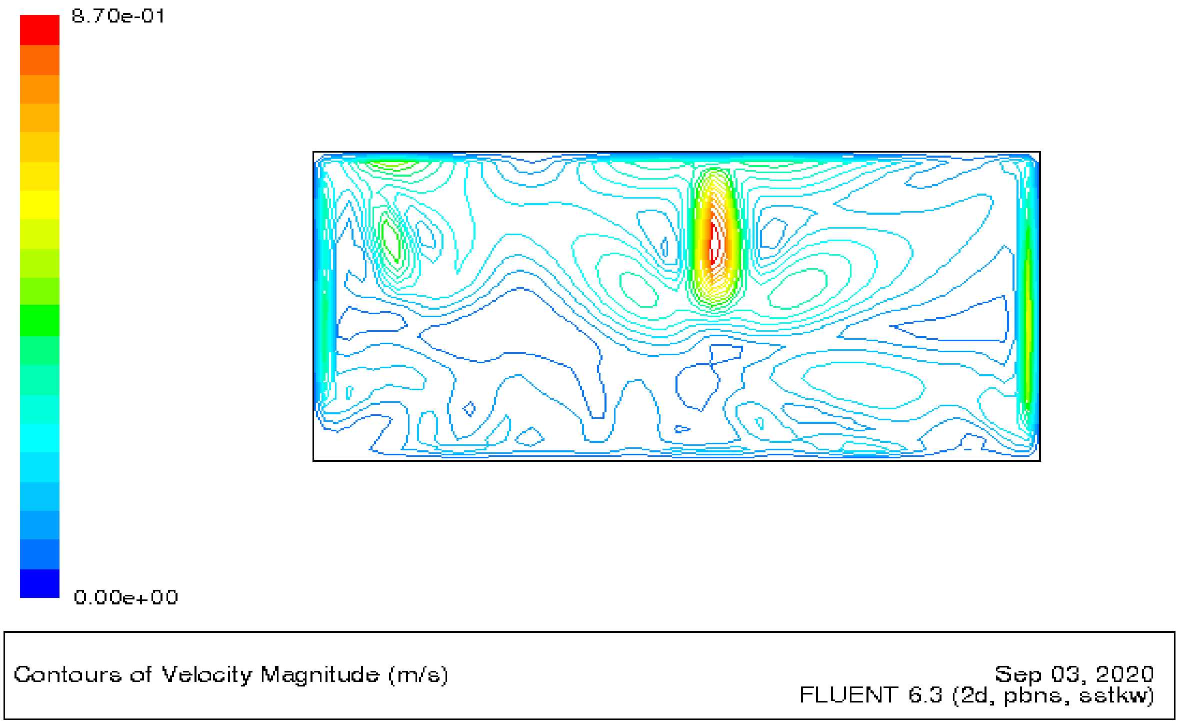


Fig 6.2.4 contours of velocity magnitude of Rayleigh number $\mathbf{10^{13}}$.



6.3 Streamline distribution

Streamline is an imaginary line in a fluid such that the tangent at any point shows the path of the speed of an element of the fluid at that point. The lowest value indicated is of the Rayleigh number 10^{10} which is 0.100kg/s followed by that of Rayleigh number 10^{11} which is 0.233kg/s. This value increases as the Rayleigh number increases as depicted by that of a Rayleigh number 10^{12} which is 0.406kg/s and the highest which is 1.310kg/s as shown by that of Rayleigh number 10^{13}. In the figure 6.3.1, the vortices are big in size and they assume a circular part which deforms as distance increases from their centres. In 6.3.2, the radius of centre circle reduces which as well decreases as the Rayleigh number increases to 10^{13} as seen in figure 6.3.3.

In 6.3.4, the two center cell deforms and takes an oval shape. The vortices become parallel as the Rayleigh number increases.

Figure 6.3.1 contours of streamline of Rayleigh number $\mathbf{10^{10}}$

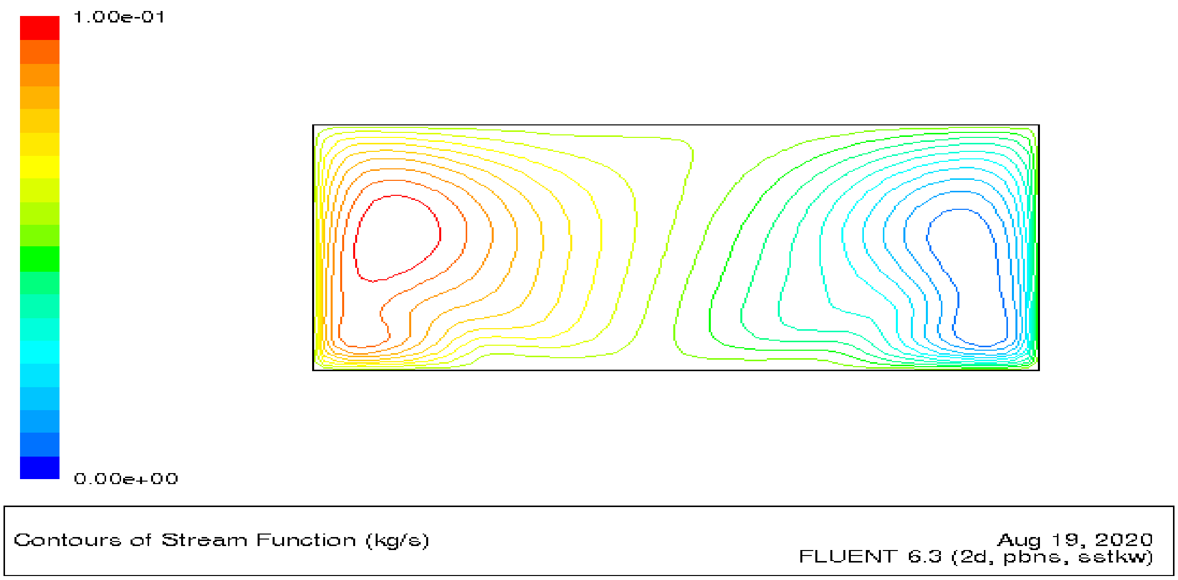


Figure 6.3.2 contours of streamline of Rayleigh number 10^{11}

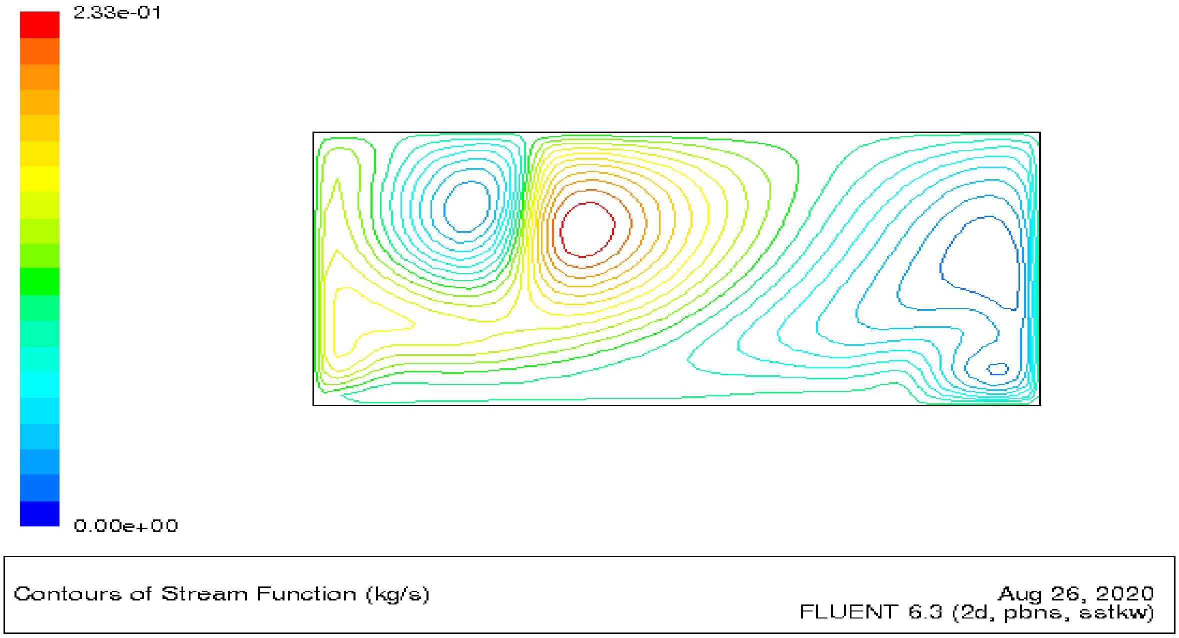


Figure 6.3.3 contours of streamline of Rayleigh number **10^{12}**

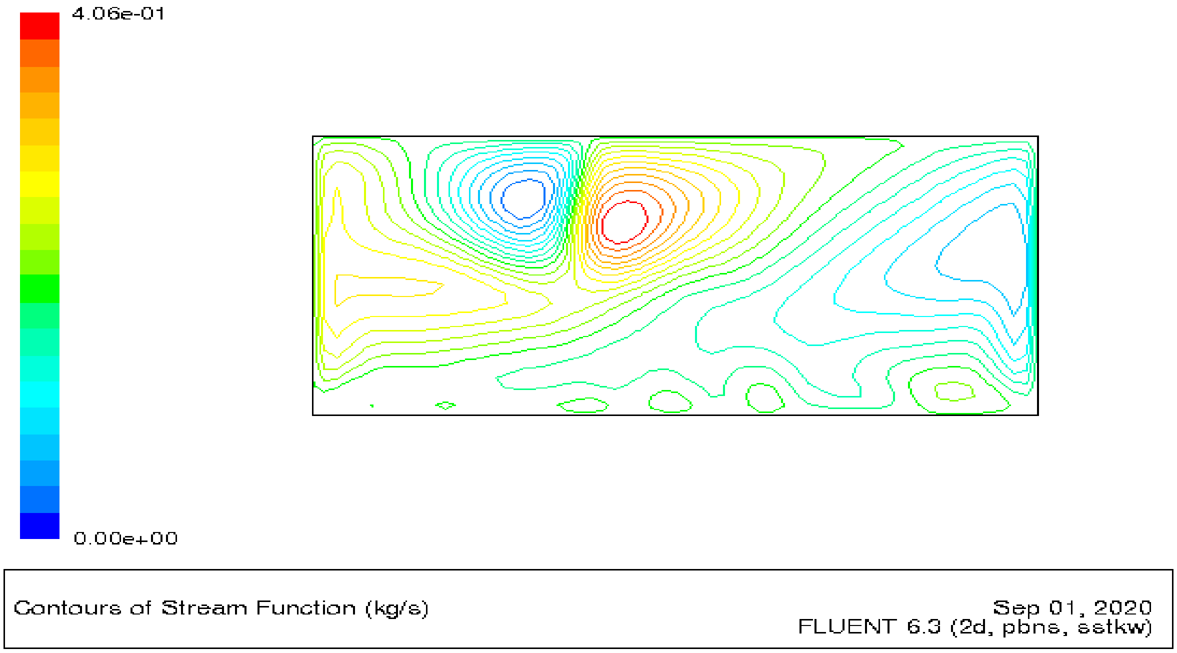
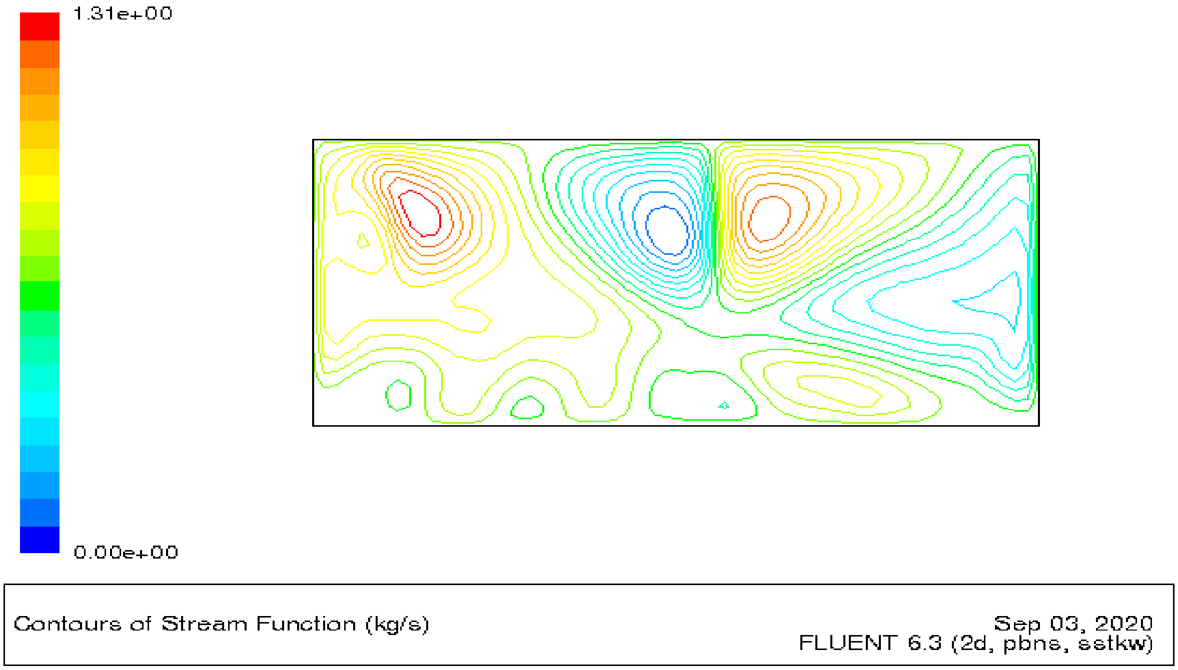


Figure 6.3.4 contours of streamline of Rayleigh number $\mathbf{10^{13}}$



6.4 Conclusion

The objective of the study was to simulate turbulent natural convention in a rectangular enclosure with delocalized heating and cooling. To achieve this, we had set up specific objectives which were achieved as follows:

Numerical data were set for k-ω-SST turbulence model. The boussinesq estimation were utilized allowing the conversation equation to be simplified. Discretization of governing equations with limit conditions were done using three-point forward and central difference approximations. Streamlines, Isotherms and Velocity magnitudes for Rayleigh numbers $10^{10}, 10^{11}, 10^{12}$ *and* 10^{13} were generated and showed that the increase in Rayleigh number decreased the turbulence.

The results showed that increased Rayleigh number decreased speed and vortices became more parallel thus decreasing turbulence. So the Rayleigh number has an important influence in temperature field and fluid stream in horizontal enclosure heated from the bottom of one of the sides.

It is also evident from the results obtained above that the position of the heater and the cooler greatly affected the distribution of heat in the enclosure. The heat distribution was found to be better when the heater and cooler were placed on the same side of the wall and it also increased with increase in Rayleigh number.

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